

ARE NEGATIVE SUPPLY SHOCKS EXPANSIONARY AT THE ZERO LOWER BOUND?

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Abstract

The standard new Keynesian model predicts that economies behave very differently at the zero lower bound: completely wasteful government spending or forward-guidance are very stimulative, and capital destruction or oil supply shocks are expansionary. I provide empirical evidence on this premise and find it wanting: The Great East Japan earthquake and oil supply shocks are contractionary at the zero lower bound. Modifications of the model that are consistent with the data also overturn other unusual policy predictions, such as large fiscal multipliers. This suggests that many of the usual rules of economics continue to hold at zero nominal interest rates.

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“As some of us keep trying to point out, the United States is in a liquidity trap: [...] This puts us in a world of topsy-turvy, in which many of the usual rules of economics cease to hold. Thrift leads to lower investment; wage cuts reduce employment; even higher productivity can be a bad thing. And the broken windows fallacy ceases to be a fallacy: something that forces firms to replace capital, even if that something seemingly makes them poorer, can stimulate spending and raise employment.”

Paul Krugman, 3rd September 2011.

1. Introduction

The standard new Keynesian model predicts that economies behave very differently at the zero lower bound: completely wasteful government spending or forward-guidance are very stimulative, capital destruction or oil supply shocks are expansionary, and reducing pricing frictions exacerbates a recession. I provide empirical evidence on this premise and find it wanting: The Great East Japan earthquake and oil supply shocks are contractionary at zero nominal interest rates.

In simple new Keynesian models (e.g., [Woodford, 2003](#); [Werning, 2012](#)) aggregate demand is determined by intertemporal substitution, and not a static old Keynesian consumption or investment function. Lower expected real interest rates encourage consumers to pull forward consumption and vice-versa. Capital destruction, higher real oil prices, or higher government spending raise marginal costs, which raises expected inflation. In normal times the central bank raises nominal interest rates more than one-for-one with inflation, raising expected real interest rates, and inducing consumers to postpone consumption. But if the central bank does not respond, for example because it is constrained by the zero lower bound, then expected real interest rates decline causing an increase in consumption and output. Thus, I look for events that raise current and future marginal costs, I check that expected inflation does rise and nominal interest rates do not, and I look for the predicted output rise.

The first event is the Great East Japan Earthquake in 2011. I verify that expected inflation rises and nominal interest rates do not, so that expected real interest rates decline. However, in contrast to the new Keynesian prediction, Japan experienced a large drop in

output following the earthquake and tsunami.

Second, I identify oil supply shocks following [Kilian \(2009\)](#) and estimate their impact on Japan at the zero lower bound. Again, expected inflation rises and nominal interest rates do not, but output falls. I also provide evidence against a weaker interpretation of the new Keynesian model: Because nominal rates do not rise at the zero lower bound, supply shocks should be less contractionary than in normal times. However, I also reject this weaker premise since oil supply shocks are, if anything, more contractionary at the zero lower bound.

I highlight modifications of the new Keynesian model that are consistent with the data ([Braun, Körber, and Waki, 2016](#); [Cochrane, 2016](#); [Kiley, 2016](#); [Mertens and Ravn, 2014](#)). These also overturn the other unusual predictions for the zero lower bound, such as large fiscal multipliers. This suggests that the zero lower bound may not be so “topsy-turvy” after all, and many of the usual rules of economics continue to hold.

Many previous papers have emphasized the unusual new Keynesian policy predictions for the zero lower bound. [Eggertsson, Ferrero, and Raffo \(2014\)](#), figures 4 and 8, stress that raising productivity can be contractionary in a new Keynesian model at the zero lower bound. [Rouilleau-Pasdeloup and Zhutova \(2015, p. 14-15\)](#) find that Hoover’s efforts to maintain high wages is expansionary in their new Keynesian model of the Great Depression. Similarly, [Eggertsson \(2012\)](#), proposition 2, argues that the National Industrial Recovery Act raised output in the Great Depression by allowing firms and workers to collude and raise mark-ups. [Christiano, Eichenbaum, and Rebelo \(2011, figure 2\)](#), [Woodford \(2011, equation 38\)](#), and [Carlstrom, Fuerst, and Paustian \(2014, figure 1\)](#), among many others, emphasize the large fiscal multipliers generated by standard new Keynesian models at the zero lower bound. According to [Eggertsson \(2011\)](#), section 7 table 3, cutting marginal tax rates on labor income would reduce output. And [Eggertsson \(2010c, proposition 2\)](#) and [Eggertsson and Krugman \(2012, p. 1486-1487\)](#) argue that a greater willingness to supply labor would reduce output. My results suggest skepticism that these predictions apply in practice.

Previous empirical examinations of supply shocks at the zero lower bound include [Multi-](#)

gan (2010, 2012), who argues that seasonal labor inflows do not appear to be contractionary and higher minimum wages do not appear to be expansionary in the data. However, Eggertsson (2010a) disputes that these are valid tests of the standard new Keynesian model, because the shocks are either forecastable or permanent and therefore do not raise inflation expectations and lower expected real interest rates. Bachmann, Berg, and Sims (2015) show that consumers expecting above-average inflation have lower willingness to spend at the zero lower bound. However, in their cross-sectional analysis they cannot test whether raising aggregate inflation expectations is expansionary at the zero lower bound as predicted by the standard new Keynesian model. My analysis is robust to these critiques, as the Great East Japan Earthquake and oil supply shocks raise aggregate inflation expectations and lower expected real interest rates, yet still are contractionary. In line with my results, Cohen-Setton, Hausman, and Wieland (2016) find that mandatory wage increases and hours reductions were also contractionary in Great Depression France.

2. Model

I use a simple new Keynesian model to convey the intuition for its unusual policy predictions at the zero lower bound, and I show that one obtains the same predictions from the medium-scale Smets and Wouters (2007) model.

I follow Werning’s (2012) continuous-time set-up,¹

$$\frac{dc(t)}{dt} = i(t) - \pi(t) - \rho \tag{1}$$

$$\frac{d\pi(t)}{dt} = \rho\pi(t) - \kappa^*[c(t) - a(t)] \tag{2}$$

where $c(t)$ is the log-linear deviation of consumption from steady state, $i(t)$ is the nominal interest rate, $\pi(t)$ is the inflation rate, and $a(t)$ is the log-linear deviation of productivity from trend. All output is consumed, so $c(t)$ is also equal to output $y(t)$. The parameter

¹His equations 1a-b, where $x(t) = c(t)$ and $a(t) = 0$ in his set-up. Equations (1)-(2) correspond to the discrete-time equations 1.12-1.13 in Woodford (2003), Ch. 4.

$\rho > 0$ is the discount rate, and $\kappa^* > 0$ is a nonlinear function of deep parameters. (See [online appendix D](#) for a derivation of equation (2).)

Equation (1) is the Euler equation, in which consumption growth (equal to output growth) is proportional to the real interest rate $i(t) - \pi(t)$ net of the discount rate ρ . Given a return to steady state, $\lim_{t \rightarrow \infty} c(t) = 0$, consumption and output are determined by the expected path of real interest rates,

$$y(t) = c(t) = - \int_{s=t}^{\infty} [i(s) - \pi(s) - \rho] ds. \quad (3)$$

Aggregate demand and output is determined by intertemporal substitution. High expected real interest rates induce a postponement of consumption, causing a fall in output, whereas low expected real interest rates raise consumption and output today.

Equation (2) is the new Keynesian Phillips curve, in which current inflation is equal to the sum of discounted expected real marginal costs of production,

$$\pi(t) = \kappa^* \int_{s=t}^{\infty} e^{-\rho(s-t)} [c(s) - a(s)] ds. \quad (4)$$

Holding consumption fixed, a decline in productivity $a(t)$ raises the marginal cost of production and thus raises inflation. But lower $a(t)$ can equivalently represent any shock that raises the marginal cost of production, such as an increase in the disutility of labor or an increase government spending. Thus, a decline in $a(t)$ can be more broadly interpreted as a negative supply shock.

For simplicity, I solve the perfect foresight version of the model, with an unexpected negative productivity shock at time zero that lasts for T periods,

$$\begin{aligned} a(t) &= \bar{a} < 0 & 0 \leq t < T, \\ a(t) &= 0 & t \geq T. \end{aligned}$$

I determine the impact of this shock under two different monetary policy regimes. In “normal times” the central bank follows an “active” interest rate rule, $i(t) = \rho + \phi\pi(t)$ where

$\phi > 1$. Thus, the nominal interest rate rises more than one-for-one with inflation. In the second regime the nominal interest rate is pegged at $i(t) = \rho$. Monetary policy is “passive” and does not respond to shocks. A likely cause of unresponsive monetary policy is the zero lower bound, but whether the nominal interest is pegged at zero or at a positive constant is unimportant to the economics of the new Keynesian model (see e.g., [Christiano et al., 2011](#), p.100-101 on this point). Directly assuming a pegged interest rate simplifies the algebra of the model relative to a more complex set-up in which a disturbance causes the zero lower bound to bind for the duration of the productivity shock. [Online appendix E](#) shows that the simple version of the model presented here and the more complex set-up yield identical predictions for the response to a productivity shock.

I follow the standard equilibrium selection principles in the new Keynesian literature. I restrict my attention to equilibria that are bounded going forward in time (see [Woodford, 2003](#), p.77-79). Also, following [Eggertsson \(2010c\)](#), [Christiano et al. \(2011\)](#), [Woodford \(2011\)](#), and [Werning \(2012\)](#) among others, I assume that the central bank can commit to a zero inflation target after the shock passes, $\pi(T) = 0$.² These two restrictions pin down a locally unique forward-bounded equilibrium around the zero-inflation steady state. [Cochrane \(2016\)](#) shows that the unusual policy predictions for the zero lower bound are a consequence of the $\pi(T) = 0$ restriction, and he argues against this equilibrium selection. I instead take the standard new Keynesian model including its equilibrium selection as given, and then examine whether the unusual policy predictions emphasized in the literature are consistent with the data.

The upper panel of [figure 1](#) plots the output and inflation responses in normal times to a negative productivity shock. The parameter values are $\rho = 0.02$, $\kappa^* = 0.35$, $\phi = 2$, $T = 4$, and $\bar{a} = -0.01$. The decline in productivity raises the marginal cost of production, which

²This boundary condition is typically implemented using an interest rate rule that has a zero inflation target and satisfies the Taylor principle. An example is $i(t) = \max\{\rho + \phi\pi(t), 0\}$ where $\phi > 1$. Then, given that all shocks have disappeared at T , $\pi(T) = 0$ on the locally unique equilibrium path that is bounded forward in time. See Proposition 1 in [Eggertsson \(2010c\)](#), [Christiano et al. \(2011\)](#) p.90-91, and [Woodford \(2011\)](#) p.17. Like [Werning \(2012\)](#), I directly impose $\pi(T) = 0$.

raises inflation. In normal times, the central bank raises nominal interest rates more than one-for-one with inflation, so expected real interest rates rise. This encourages consumers to postpone their consumption today and demand falls. Thus, output jumps down on the news of low productivity and then gradually converges to steady state. For general parameter choices the same intuition applies, and output will always decline along with productivity,

$$y(t) = c(t) = \frac{1}{\mu_1 - \mu_2} \left[\mu_1(1 - e^{-\mu_2(T-t)}) - \mu_2(1 - e^{-\mu_1(T-t)}) \right] \bar{a} < 0$$

$$\pi(t) = \frac{\kappa^*}{\mu_1 - \mu_2} (e^{-\mu_1(T-t)} - e^{-\mu_2(T-t)}) \bar{a},$$

for $0 \leq t < T$, where μ_1, μ_2 are the eigenvalues

$$\mu_1 = \frac{\rho}{2} + \frac{\sqrt{\rho^2 - 4\kappa^*(\phi - 1)}}{2}, \quad \mu_2 = \frac{\rho}{2} - \frac{\sqrt{\rho^2 - 4\kappa^*(\phi - 1)}}{2}.$$

The algebra steps for this and the following derivation are in [appendix A](#).

For the same parameter values, the lower panel of [figure 1](#) displays the output and inflation response under an interest rate peg, or, equivalently, at the zero lower bound. The negative productivity shock still raises current and future inflation, but now nominal interest rates are unchanged. Therefore, expected real interest rates fall, inducing consumers to consume now rather than later. Thus, output jumps up on news of lower productivity and then gradually declines. In line with this intuition, output and inflation rise for any parameter values,

$$y(t) = c(t) = \frac{1}{\lambda_1 - \lambda_2} \left[\lambda_1(1 - e^{-\lambda_2(T-t)}) - \lambda_2(1 - e^{-\lambda_1(T-t)}) \right] \bar{a} > 0$$

$$\pi(t) = \frac{\kappa^*}{\lambda_1 - \lambda_2} (e^{-\lambda_1(T-t)} - e^{-\lambda_2(T-t)}) \bar{a} > 0,$$

for $0 \leq t < T$ where λ_1, λ_2 are the eigenvalues

$$\lambda_1 = \frac{\rho}{2} + \frac{\sqrt{\rho^2 + 4\kappa^*}}{2} > 0, \quad \lambda_2 = \frac{\rho}{2} - \frac{\sqrt{\rho^2 + 4\kappa^*}}{2} < 0.$$

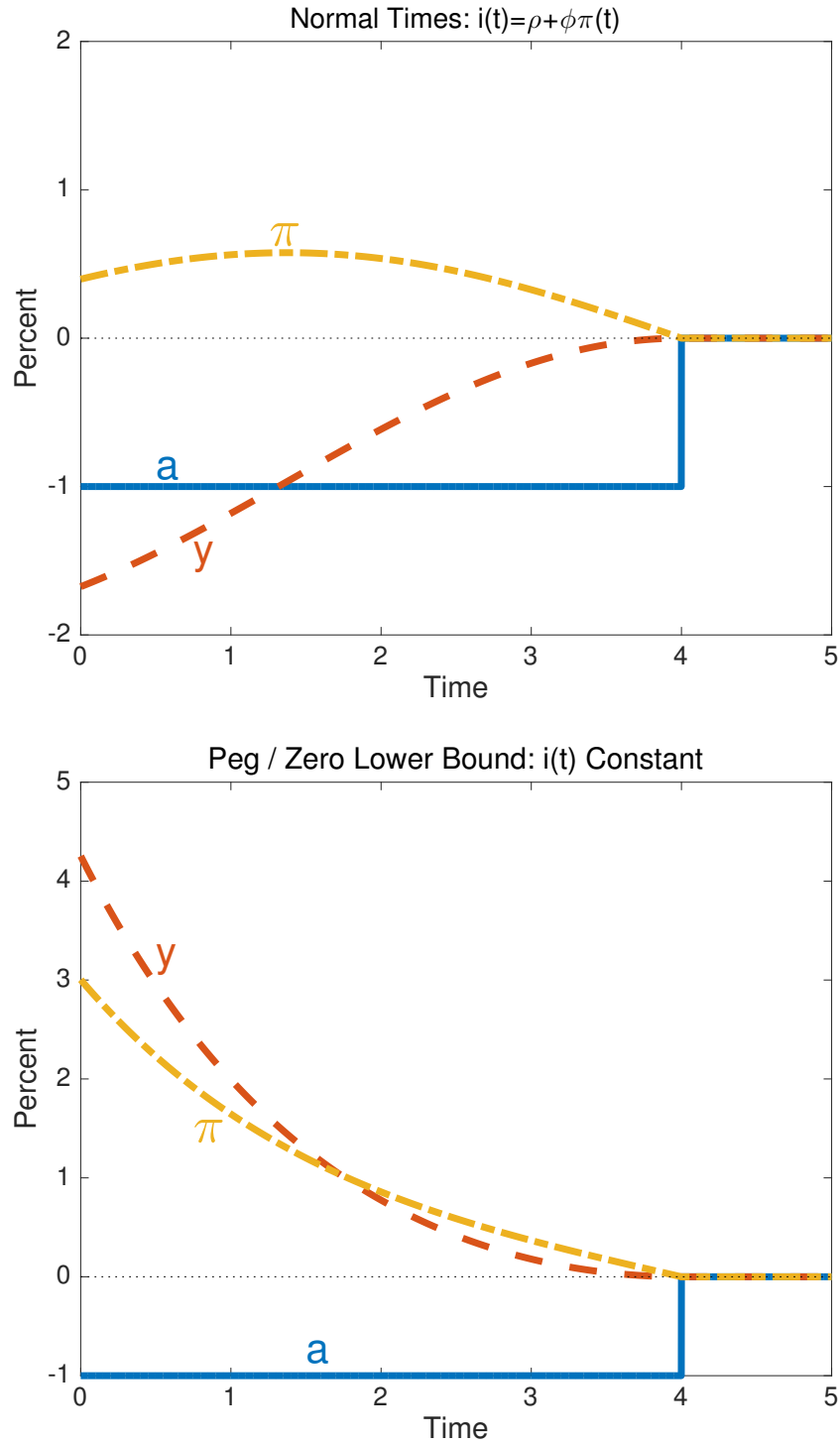


Figure 1 – Impulse response function for a negative productivity shock in the standard new Keynesian model. The parameter values are $\rho = 0.02$, $\kappa^* = 0.35$, $\phi = 2$, $T = 4$, and $\bar{a} = -0.01$. The top panel shows the impact in normal times, where the central bank follows an “active” monetary policy rule, $i(t) = \rho + \phi\pi(t)$ with $\phi > 1$. The bottom panel shows the impact under an interest rate peg, or equivalently, the zero lower bound, so the nominal interest rate is constant. Both panels impose the standard new Keynesian equilibrium selection criteria.

I now add government spending, $g(t)$, to the model,

$$\begin{aligned}\frac{dc(t)}{dt} &= i(t) - \pi(t) - \rho \\ \frac{d\pi(t)}{dt} &= \rho\pi(t) - \kappa^*[c(t) - \psi_a a(t) + \psi_g g(t)],\end{aligned}$$

where $\psi_a, \psi_g > 0$. By symmetry, a positive government spending shock, $g(t) > 0$, has the same effect as a negative productivity shock $a(t) = -\frac{\psi_g}{\psi_a}g(t) < 0$. Thus, government spending only crowds in consumption (generating a multiplier above 1) if lower productivity is also expansionary. *In the standard new Keynesian model testing for expansionary negative supply shocks is the same as testing for a large fiscal multiplier.*

Medium-scale new Keynesian models make similar predictions, because intertemporal substitution remains the central propagation mechanism (Kaplan, Moll, and Violante, 2016). I use the ubiquitous Smets and Wouters (2007) model for illustration. As before, I study an unexpected, negative productivity shock at $t = 0$. In appendix B, I provide detailed instructions for constructing the impulse response functions.

The top panel of figure 2 plots the result for normal times. As in the simple model, the negative productivity shock causes higher inflation, inducing the central bank to raise real interest rates. Agents postpone consumption and investment, and output falls.

For the passive monetary policy regime, I choose an interest rate peg that lasts for $T = 100$ quarters. This implies that almost all of the productivity shock occurs at constant nominal interest rates, similar to the simple model. $T = 100$ is also consistent with the persistence of the zero lower bound in Japan. The bottom panel of figure 2 plots the result. Now the increase in inflation lowers expected real interest rates. Agents pull forward consumption and investment, and output expands significantly.

I now turn to empirically testing the prediction that negative supply shocks are expansionary under passive monetary policy. I look for negative supply shocks that raise expected inflation, but do not raise nominal interest rates (so monetary policy remains passive). I then check if output expands as predicted by the standard new Keynesian model.

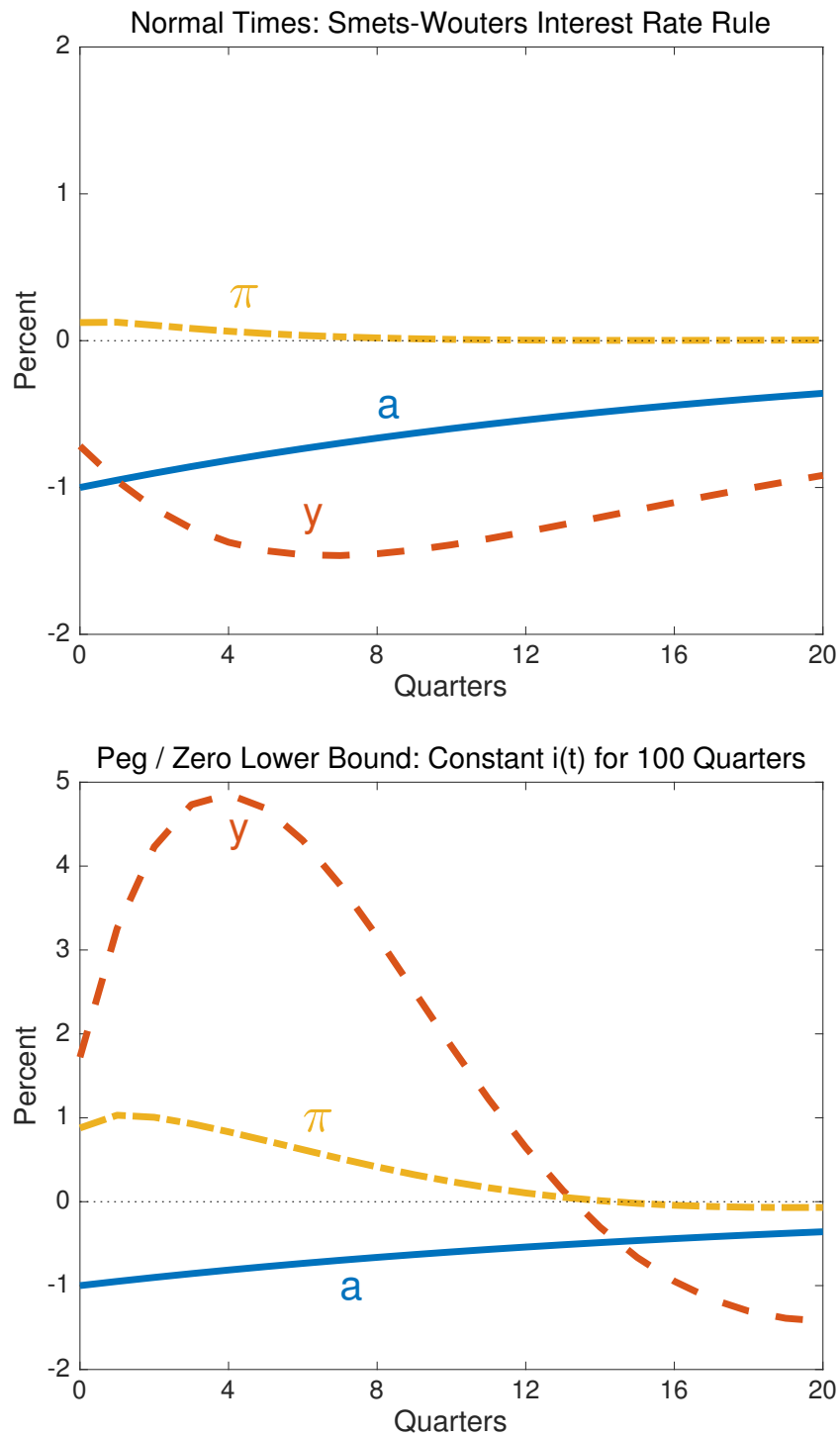


Figure 2 – Impulse response function for a -1% negative productivity shock in the [Smets and Wouters \(2007\)](#) model. The top panel shows the impact in normal times, where the central bank follows the estimated interest rate rule in [Smets and Wouters \(2007\)](#). The bottom panel shows the impact for an interest rate peg, or, equivalently, the zero lower bound, where the nominal interest rate is constant for $T = 100$ quarters.

3. The Great East Japan Earthquake

On Friday March 11th 2011, a magnitude-9.0 earthquake off the east coast of Japan triggered a tsunami that caused extensive damage to structures, created an electricity shortage, and disrupted global supply chains. To connect this shock to the analysis above, I add an exogenous capital stock $K(t)$ to the standard new Keynesian model. The production function is now $Y(t) = A(t)K(t)^\alpha L(t)^{1-\alpha}$, where $L(t)$ is labor input. By symmetry, an earthquake destroying capital $K(t)$ is just like a decline in productivity $A(t)$. It raises the marginal cost of production and raises inflation. If the central bank does not react because it is constrained by the zero lower bound, then real interest rates fall, which stimulates consumption and output through intertemporal substitution.

While instructive, this setting does not incorporate any endogenous investment response. I therefore also simulate a capital destruction shock in the [Smets and Wouters \(2007\)](#) model. I construct impulse response functions as detailed in [appendix B](#), and I plot them in [figure 3](#). In normal times destroying 1% of the capital shock generates a tiny expansion followed by a persistent contraction. But if nominal interest rates are constant, then output expands, by almost 2%, and the destroyed capital is quickly replenished. The Great East Japan Earthquake provides a testing ground for this prediction since Japan has had essentially constant zero nominal interest rates since 2009.

The available evidence suggests that the earthquake was contractionary. Industrial production declined by 6.4% from February to March 2011. Between the fourth quarter of 2010 and the first quarter of 2011, real output declined by 2.0% and real consumption contracted by 1.8%. Japan only recovered to its pre-earthquake GDP peak by the first quarter of 2012.

I can also proxy for pre-existing trends using professional forecasts. Before the earthquake, in February 2011, Consensus Economics predicted GDP to grow by 1.5% from 2010-2011 ([figure 4\(a\)](#)). The actual GDP growth rate was -0.4%. In the aftermath of the earthquake, in April 2011, the Consensus Economics forecast for 2011 GDP growth were revised

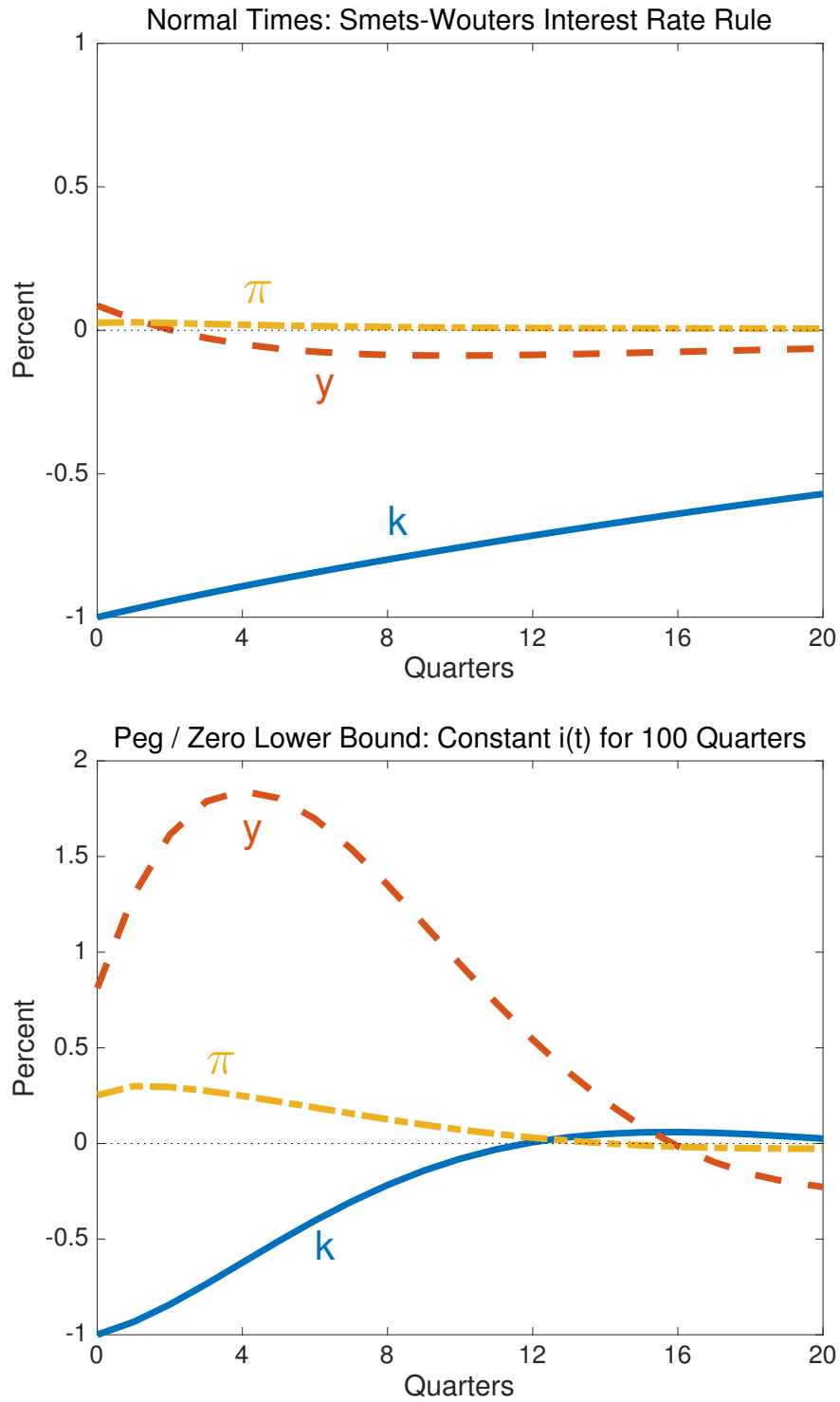
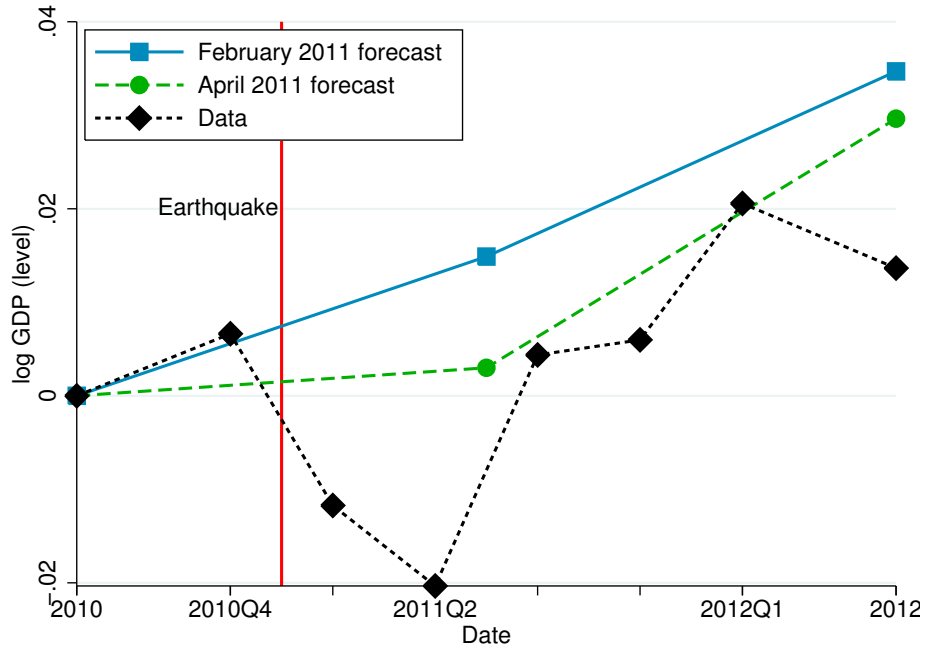
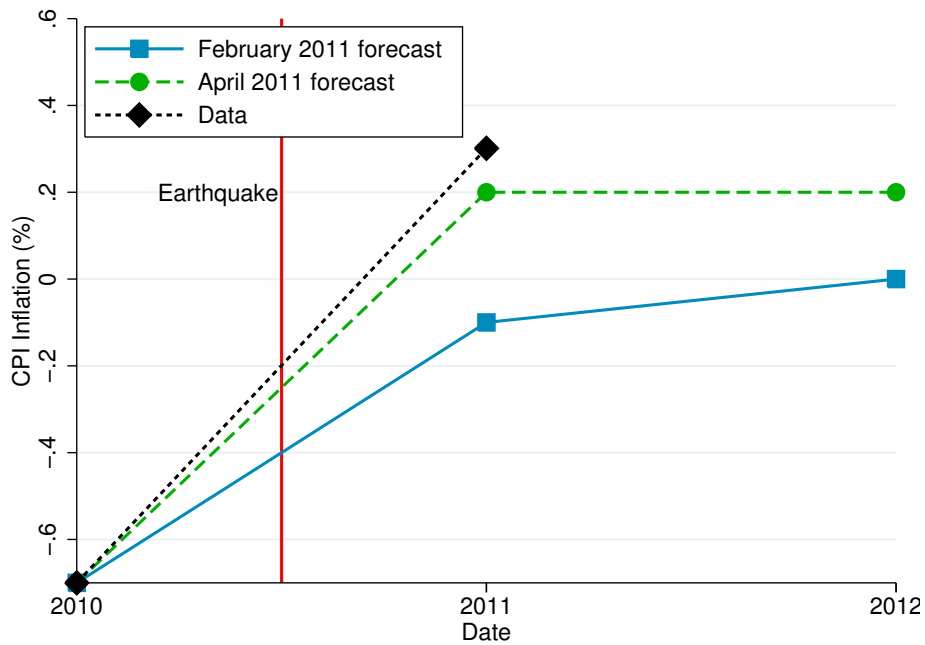


Figure 3 – Impulse response function for a -1% capital destruction shock in the [Smets and Wouters](#) model. The top panel shows the impact in normal times, where the central bank follows the estimated interest rate rule in [Smets and Wouters \(2007\)](#). The bottom panel shows the impact for the zero lower bound regime, where the nominal interest rate is constant for $T = 100$ quarters.



(a) GDP



(b) Inflation

Figure 4 – Consensus Economics forecasts from before the Great East Japan Earthquake (February 2011) and after (April 2011). Forecasts are for annual GDP and year-on-year inflation. GDP data are annual for 2010 and 2012 and quarterly from 2010Q4 until 2012Q1. CPI data are annual year-on-year inflation. Red vertical lines indicate the earthquake date, March 11, 2011.

down by 1.2 percentage points. Overall, the sharpness of the contraction and the forecast revisions suggest a causal effect of the earthquake.

The contraction was accompanied by an increase in inflation expectations. Japanese consensus inflation forecasts for 2011 and 2012 rose following the earthquake by 0.3 percentage points and 0.2 percentage points respectively (figure 4(b)).³ In addition, the 10-year inflation swap rate rose from an average of -3 basis points from March 7th through 10th to an average of +3 basis points from March 14th through 18th.⁴

It is clear that Japan was at the zero lower bound at the time; the Bank of Japan discount rate was at 0.3% and the uncollateralized overnight call rate was below 0.1%. Of course, the output expansion in the standard new Keynesian model and the Smets and Wouters model is predicated on the assumption that expected nominal interest rates also do not rise. The Japanese experience is consistent with this assumption. The yield on 10-year government bonds fell from an average of 1.30% from March 7th through 10th to 1.22% from March 14th through 18th.⁵ Twenty-year and 30-year bond yields also declined.

In short, the Great East Japan Earthquake and tsunami was a negative supply shock at the zero lower bound. Inflation expectations rose and nominal interest rates did not rise, but the Japanese economy contracted contrary to the prediction of the standard new Keynesian model.

A potential concern is that the Great East Japan Earthquake may not be a temporary shock, as assumed in the simple new Keynesian model or the Smets and Wouters model, where any destroyed capital gets rebuilt. However, Brueckner (2014) estimates that the capital stock returns to steady state 20 years after an earthquake. Further, the April survey's GDP growth forecast for 2012 was revised *upward*, making up half of the loss from the

³I use February and April forecasts, because the March forecasts were released only shortly after the earthquake so that some of them were outdated (e.g., the Morgan Stanley forecast).

⁴Data on inflation swaps and bond yields are from Bloomberg.

⁵One reason for the decline in nominal interest rates was a Bank of Japan announcement to increase its Asset Purchase Program in response to the earthquake and tsunami. I attribute this response to the earthquake and tsunami in the sense that the monetary easing would not have occurred otherwise. Of course, from the perspective of the standard new Keynesian model, it becomes more puzzling that a negative supply shock and a concurrent monetary expansion do not raise output at the zero lower bound.

forecast revision for 2011, as shown in [figure 4\(a\)](#). This suggests that the Japanese economy is catching up to its balanced growth path. The International Monetary Fund also did not change its estimates for potential output in Japan ([Hausman and Wieland, 2014](#)). Finally, to the extent that a potential phase-out of nuclear power in Japan merely pulls forward an eventual phase-out and/or substitution with alternative technologies, then that effect should also be viewed as temporary. In the event, with the first nuclear reactors returning to the grid in late 2015, the abandonment of nuclear power appears to have been only a temporary policy change. This suggests that it is reasonable to treat the earthquake and tsunami as a temporary shock.

4. Oil Supply Shocks

I next study global oil supply shocks. In a simple new Keynesian model with oil, these shocks affect consumption and inflation through the log-linearized real price of oil $p^o(t)$,

$$\begin{aligned}\frac{dc(t)}{dt} &= i(t) - \pi(t) - \rho \\ \frac{d\pi^y(t)}{dt} &= \rho\pi^y(t) - \kappa^*[c(t) + \psi p^o(t)] \\ \pi(t) &= \pi_t^y + \gamma \frac{dp^o(t)}{dt}.\end{aligned}$$

π^y is the inflation rate of the non-oil good, $\psi > 0$ captures the use of oil in that good's production, and γ is the steady state share of oil in consumption. A higher real price of oil raises the marginal cost of production, and it also has a direct effect on consumer price inflation. But the same intuition from a negative productivity shock also applies to a temporary rise in the real price of oil, $p^o(t) > 0$ for $0 \leq t < T$. If the increase in real oil prices raises inflation expectations, and the central bank does not react by raising nominal interest rates, then consumption and GDP will rise through intertemporal substitution. In [online appendix F](#), I verify that this conclusion also holds in a fully spelled-out new Keynesian open economy model with oil imports.

I focus on oil supply shocks as primitive shocks raising real oil prices, because global demand shocks that raise real oil prices are likely correlated with domestic demand shocks. I follow [Kilian's \(2009\)](#) identification strategy for uncovering oil supply shocks. He estimates a VAR with three monthly variables: the growth in global oil production, $\Delta prod_t$, a measure of global real economic activity, rea_t , and log real oil prices, rpo_t .⁶ I denote the data vector by $x_t = (\Delta prod_t, rea_t, rpo_t)'$. The structural VAR representation in [Kilian \(2009\)](#) is,

$$A_0 x_t = \alpha + \sum_{j=1}^{24} A_j x_{t-j} + \varepsilon_t. \quad (5)$$

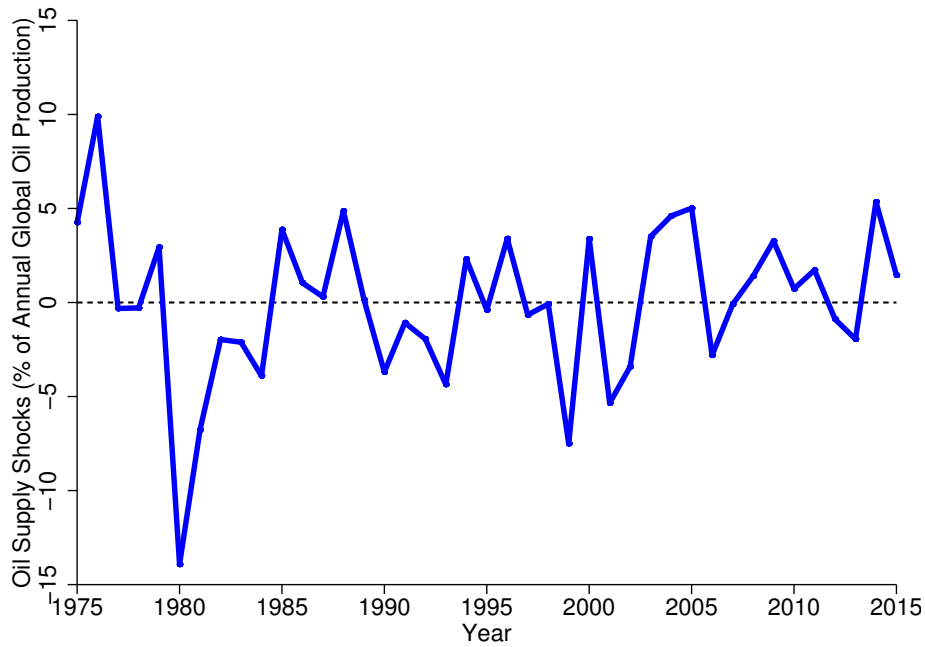
The first element in ε_t is the shock to global oil production, which I refer to as an oil supply shock. Like [Kilian \(2009\)](#) I assume that oil production responds to other structural shocks (e.g., demand shocks) with at least a one-month delay. [Anderson, Kellogg, and Salant \(2014\)](#) provide evidence for this restriction from micro data. Thus, the first row of A_0 is a unit-vector $(1, 0, 0)$. The estimated oil supply shocks are then the OLS-residuals from the first equation in (5).⁷

The sample period is February 1973 through September 2015. I plot the estimated oil supply shocks in [figure 5\(a\)](#) aggregated to an annual frequency. In [figure 5\(b\)](#) I plot the impact of an oil supply shock on real oil prices along with 95% error bands. Similar to Kilian's original series, a one-standard-deviation negative oil supply shock raises real oil prices by just over 1% after 6 months. The impulse response function is somewhat noisy, but the increase in real oil prices is statistically significant on impact. The impulse response function also converges to zero, so the negative supply shock is temporary as I assume in the theory.⁸

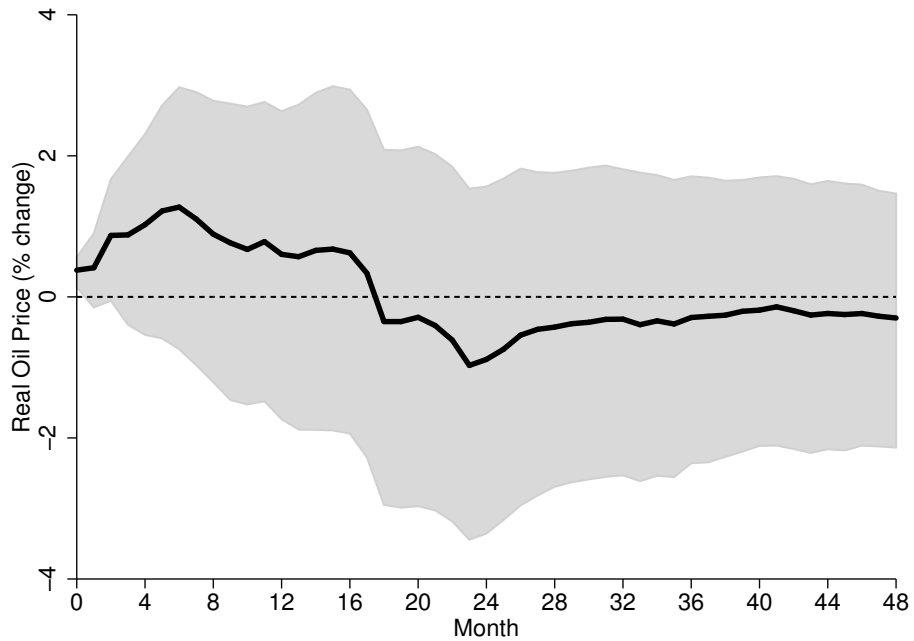
⁶Data sources are: $prod_t$ is crude oil including lease condensate from the U.S. Energy Information Administration ([link](#)). rea_t is a detrended index of shipping rates available from Lutz Kilian's website ([link](#)). rpo_t is the U.S. refiner acquisition cost of an oil barrel from the U.S. Energy Information Administration ([link](#)) deflated by the U.S. CPI. I merge data since 1/2008 with Lutz Kilian's original dataset ([link](#)).

⁷An implicit identification assumption is that news of oil demand shocks arrive within the period. If oil demand shocks for the next month are known, then producers may adjust production simultaneously with the change in demand. In that case, the VAR may attribute forecastable demand shocks to oil supply shocks. In [online appendix G](#) I therefore test if the oil supply shocks are forecastable based on past changes in oil futures prices. I do not find evidence of predictability, consistent with the identification assumption.

⁸The confidence interval also shrinks towards zero as I extend the horizon.



(a) Oil-supply shocks



(b) Impulse response of real oil prices to a negative oil supply shock.

Figure 5 – Top panel: Oil supply shocks aggregated to an annual frequency. Derived as the residuals from the first equation of the VAR (5) following Kilian (2009). Positive values show upward surprises, i.e., a positive oil supply shock. Bottom panel: Response of real oil prices to a one monthly standard deviation negative oil supply shock in the VAR. 95% confidence bands are constructed based on a recursive wild bootstrap.

I estimate responses to these oil supply shocks in Japan.⁹ To separate active and passive monetary policy regimes, I partition the sample into “normal times” and the zero lower bound period. The starting date of the normal-times regime is January 1986. [Kuttner and Posen \(2004\)](#) argue that the Bank of Japan conducted active monetary policy after this date. Normal times end and the zero lower bound begins in October 1995, the first full month after the Bank of Japan lowered the call rate to 50 basis points. This dating is in line with [Krugman \(1998\)](#), [Eggertsson and Woodford \(2003\)](#), [Svensson \(2006\)](#), and [Eggertsson \(2008\)](#). [Kuttner and Posen \(2004\)](#) identify three deflationary shocks in 1996 and 1997 that did not cause a further reduction in interest rates, suggesting that monetary policy was passive at the time. I follow [Eggertsson and Pugsley \(2006\)](#) and [Eggertsson \(2010b\)](#) and let the zero lower bound spell end in June 2006, the month before the Bank of Japan raised its policy rate to 25 basis points. Japan’s second zero lower bound spell begins in January 2009, the first full month after the Bank of Japan reduced its policy rate from 30 to 10 basis points. These dates amount to using 50 basis points as a cut-off for the zero lower bound before 1998 and 25 basis points thereafter.

For each regime, I estimate the effect of oil supply shocks on outcomes using an autoregressive distributive lag equation,

$$\Delta y_t = \alpha_x + \sum_{j=1}^m \beta_{x,j} \Delta y_{t-j} + \sum_{j=0}^k \gamma_{x,j} oil_{t-j} + \eta_t, \quad (6)$$

where $x \in \{\text{zero lower bound, normal times}\}$ indexes the regime. I use the Akaike information criterion to select lag lengths m and k . The specification implies that the impact of a past oil supply shock on current outcomes is determined by the current regime, which is consistent with the standard new Keynesian model.

I construct impulse response functions from the coefficients $\{\beta_{x,j}, \gamma_{x,j}\}_j$ for each regime x . Thus, the impulse response functions are conditional on the same regime over the horizon and do not incorporate transitions to the other regime. This is appropriate for testing if

⁹[Fukunaga, Hirakata, and Sudo \(2011\)](#) and [Iwaisako and Nakata \(2015\)](#) also study the impact of oil supply shocks on Japan, but do not distinguish between the zero lower bound and normal times.

negative supply shocks are expansionary at the zero lower bound. However, my impulse response functions are not estimates of the expected path of y given a shock, which would have to incorporate expected transitions between regimes.¹⁰

I first examine the response of inflation expectations to oil supply shocks. The inflation expectations data are from Consensus Economics. They have released inflation forecasts at quarterly frequency since 1991, typically in March, June, September and December. For each forecast release date, I sum the current and two lags of oil supply shocks to create a quarterly oil supply shock. For example, if the forecast is released in March, then the corresponding quarterly oil supply shock is the sum of the monthly oil supply shock from January, February and March. I use the four-quarter-ahead-inflation forecast for which next quarters (expected) price level is the basis. Thus, the forecast from March measures expected price changes from the second quarter of this year until the second quarter of next year. This ensures that my results are not driven by a time aggregation bias.

[Figure 6\(a\)](#) plots the dynamic impulse response function of inflation expectations to a one-standard-deviation negative oil supply shock for both the zero lower bound period and normal times. The shaded area is the 95% confidence interval for the zero lower bound. I use [Driscoll and Kraay \(1998\)](#) standard errors, which are robust to heteroscedasticity and serial correlation, and correct them for the first-stage sampling error following [Murphy and Topel \(2002\)](#). [Figure 6\(a\)](#) shows that a negative oil supply shock raises inflation expectations at the zero lower bound, with a statistically-significant peak effect of 13 basis points after four quarters. I do not find a corresponding statistically-significant increase in inflation expectations in normal times. However, I also cannot reject that the difference between normal-times and the zero lower bound reflects sampling variance.

[Figure 6\(b\)](#) displays the response of unemployment to the same shock. At the zero lower bound the unemployment rate increases, and reaches a peak response of 9 basis points after ten months. The increase is statistically significant at conventional levels. It then dissipates

¹⁰[Hayashi and Koeda \(2016\)](#) study QE in Japan with an endogenous-regime-switching SVAR.

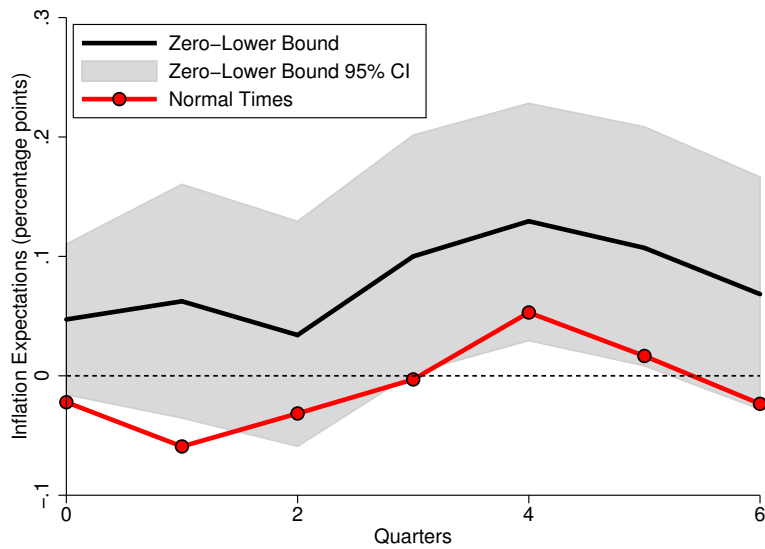
and becomes statistically insignificant twelve months after the shock. In normal times the peak unemployment response is only three basis points and marginally significant.

In [figure 6\(c\)](#) I plot the estimated effect on industrial production. At the zero lower bound industrial production contracts sharply, reaching a statistically-significant peak decline of 0.6% after two months. In the following months industrial production recovers and becomes statistically insignificant. I do not find evidence of a similar contraction in normal times. But the confidence bands for the zero lower bound and normal times do overlap, so I cannot reject that the responses are the same.

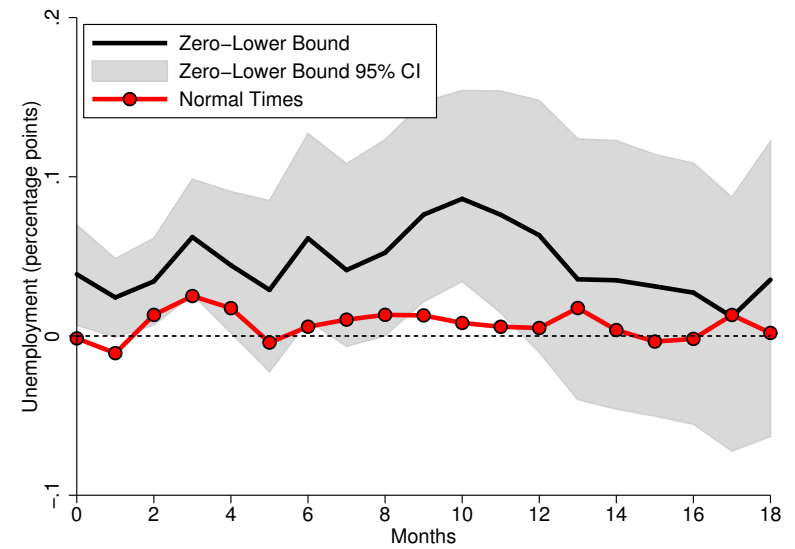
[Figure 6\(d\)](#) shows the impulse response function for monthly consumption expenditure.¹¹ This series is volatile and so are the impulse response functions. Nevertheless, at the zero lower bound, both the decline in consumption expenditures and the cumulative effect (the area under the impulse response function) are significant at the 90% level. There is only weak evidence for a decline in consumption expenditures during normal times.

Last, I verify that the oil supply shocks do not raise current and expected nominal interest rates at the zero lower bound. I therefore estimate equation (6) using changes in nominal bond yields as outcome variables. I use 12-month changes, which are less noisy than one-month changes. In [figure 7](#) I plot the response of the bond yields on news of an oil supply shock. For the zero lower bound I estimate that nominal bond yields at all maturities exhibit a *decline* after an oil supply shock at the zero lower bound. For normal times, in contrast, there is evidence that expected future nominal interest rates first decline and then increase at longer maturities.

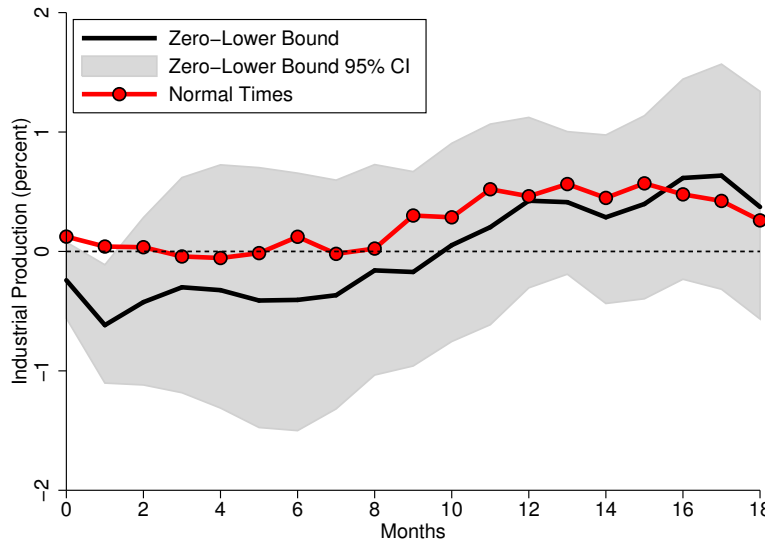
¹¹I use the real consumption expenditure index for “Two-or-more-person Households” from the Family Income and Expenditure Survey ([link](#)).



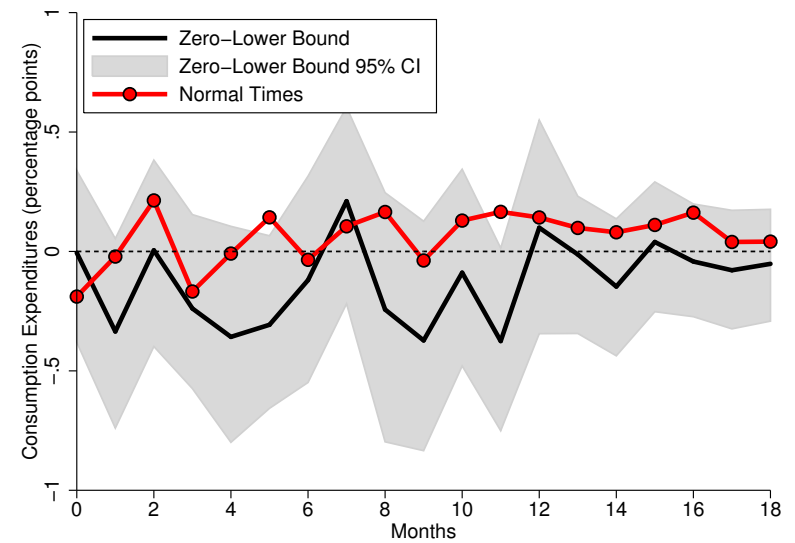
(a) Inflation Expectations



(b) Unemployment



(c) Log Industrial Production



(d) Log Consumption Expenditures

Figure 6 – Impulse response functions to negative oil supply shocks in Japan. The zero lower bound regime is 10/1995 through 6/2006 and 1/2009 through 9/2015. The normal times regime are all other months from 1/1986 through 9/2015. Impulse response functions are constructed from estimates of autoregressive distributive lag equation (6) in changes or growth rates and aggregated to levels. 95% confidence intervals are constructed by parametric bootstrap on a heteroscedasticity and autocorrelation-robust covariance matrix and corrected for the presence of estimated regressors following [Murphy and Topel \(2002\)](#). Lag lengths are $m = 0$ and $k = 6$ for inflation expectations, $m = 36$ and $k = 18$ for unemployment, $m = 0$ and $k = 18$ for industrial production, and $m = 36$ and $k = 12$ for consumption expenditures. See [section 4](#) for details.

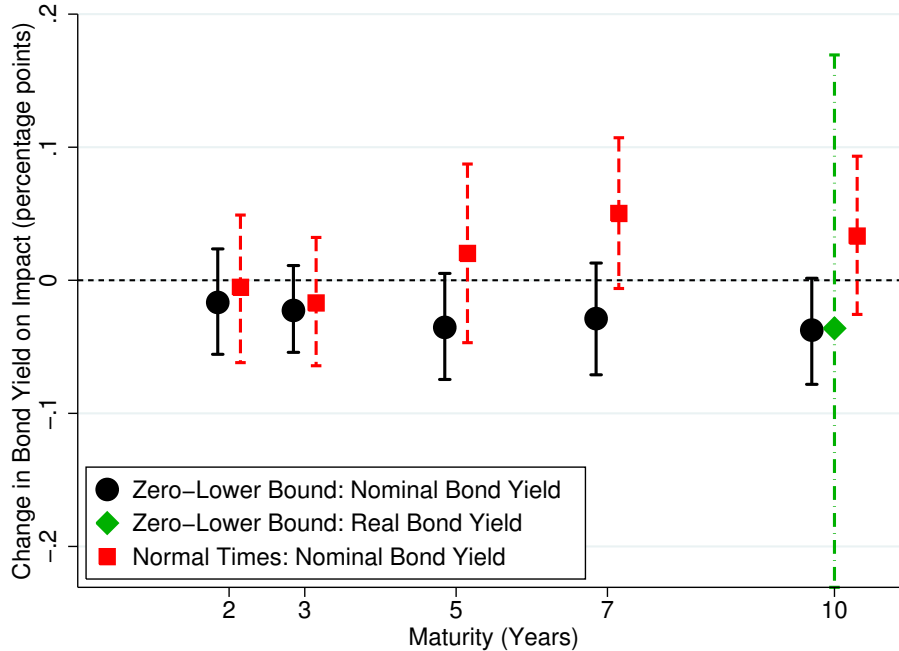


Figure 7 – Change in nominal and real bond yields in Japan after a negative oil supply shock. The zero lower bound regime is 10/1995 through 6/2006 and 1/2009 through 9/2015. Normal times are all other months from 1/1986 through 9/2015. The plotted estimates are the impact coefficients $\gamma_{x,0}$ from the autoregressive distributed lag equation (6). Lag lengths are $m = 36$ and $k = 24$. 95% error bands are constructed by parametric bootstrap on a heteroscedasticity and autocorrelation-robust covariance matrix and corrected for the presence of estimated regressors following [Murphy and Topel \(2002\)](#). See [section 4](#) for details.

The decline in expected nominal interest rates at the zero lower bound and the increase in expected inflation combine to lower expected real interest rates, which are the key quantity setting off the intertemporal consumption boom in the new Keynesian model. And for a limited sample I can directly estimate the effect of oil supply shocks on expected real interest rates. Bloomberg has data for the 10-year real bond yield in Japan from April 2004 through March 2009 and again starting in October 2013. I also construct a synthetic real bond from 10-year nominal bond yields and 10-year swap rates, which are available from March 2007 through today. For the overlap period, I use synthetic real bond yields rather than actual real bond yields because the former behave less erratically. With these data, I estimate that 10-year real bond yields decline by roughly the same amount as nominal bond yields. This suggests that long-run nominal rates largely capture the long-run real interest rate response

to oil supply shocks (figure 7).

Thus, my empirical results are inconsistent with the prediction of the standard new Keynesian model. Negative oil supply shocks are contractionary at the zero lower bound, even though expected inflation rises and nominal interest rates do not. In appendix C I show that this result is robust to different estimation equations (6), sample periods, or VAR specifications (5).

4.1. Zero lower bound vs normal times

A weaker interpretation of the standard new Keynesian model is that negative supply shocks are less contractionary at the zero lower bound. In normal times the central bank raises nominal and real interest rates, which is contractionary, but it does not do so at the zero lower bound. However, the impulse response functions in figure 6, if anything, show the opposite pattern: the contraction is typically larger at the zero lower bound than in normal times. This occurs even though expected future nominal interest rates rise more in normal times. While the point estimates show a larger contraction at the zero lower bound, the confidence intervals for both regimes typically overlap, so one cannot reject that they are equal. But even an equal output response is evidence against this weaker prediction of the standard new Keynesian model.

The only variable that responds with a different sign across the regimes is the long-term nominal bond yield, which rises in normal times and falls at the zero lower bound. One interpretation is that the increase in normal times reflects the Taylor principle. Consistent with this hypothesis, the evidence for higher long-term nominal bond yields weakens considerably when I include data before 1986. (Including these earlier data has, however, little effect on the impulse response functions for real outcomes.) At the zero lower bound, by contrast, the central bank may be more willing to accommodate an oil supply shock, because it causes a more severe contraction. A second hypothesis is that the differences simply reflect sampling error. Either way, the zero lower bound estimates are not moving in the direction predicted

by the standard new Keynesian model.

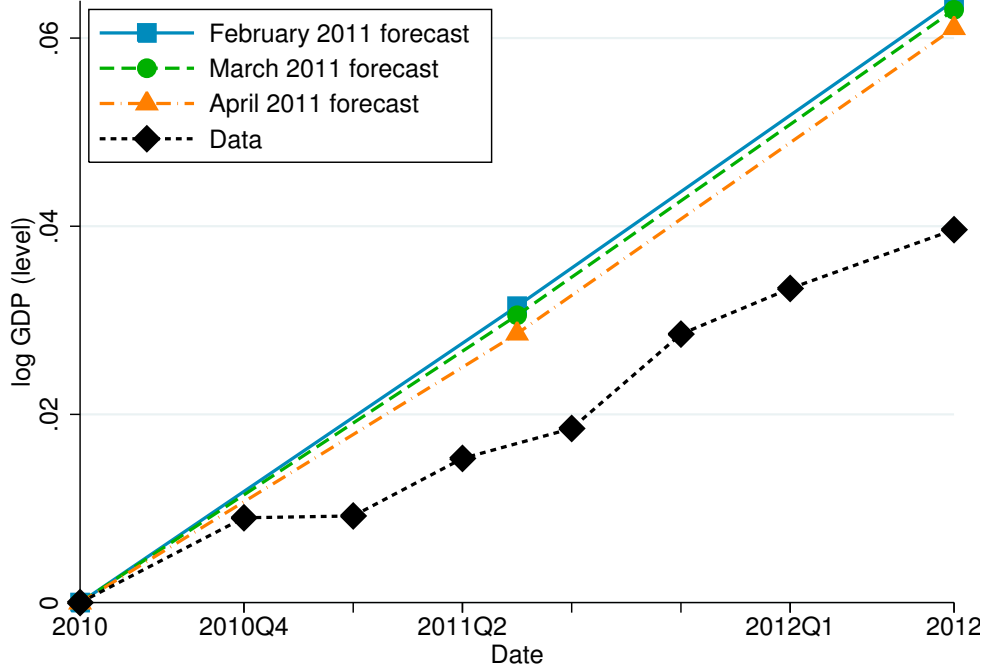
4.2. Auxiliary evidence

As corroborating evidence for the time-series methodology above, I conduct an event study around the Libyan civil war.¹² Before the civil war, Libya produced approximately 2% of global oil supply. The beginning of the civil war is typically dated on February 15th 2011. Foreign intervention officially commenced on March 19th. This conflict caused a significant contraction in Libyan oil production, which by April had declined by almost 90% relative to pre-war levels. Thus, the Libyan civil war constituted a relatively large and exogenous shock to global oil production. Consistent with this interpretation, the [Kilian \(2009\)](#) series also displays a cumulative 1.1 standard deviation oil supply shock over February 2011 and March 2011. However, while this oil supply shock is plausibly exogenous, to the extent that foreign governments increase military spending it may be correlated with a positive demand shock. In that case, this event study is biased against finding contractionary effects from negative supply shocks.

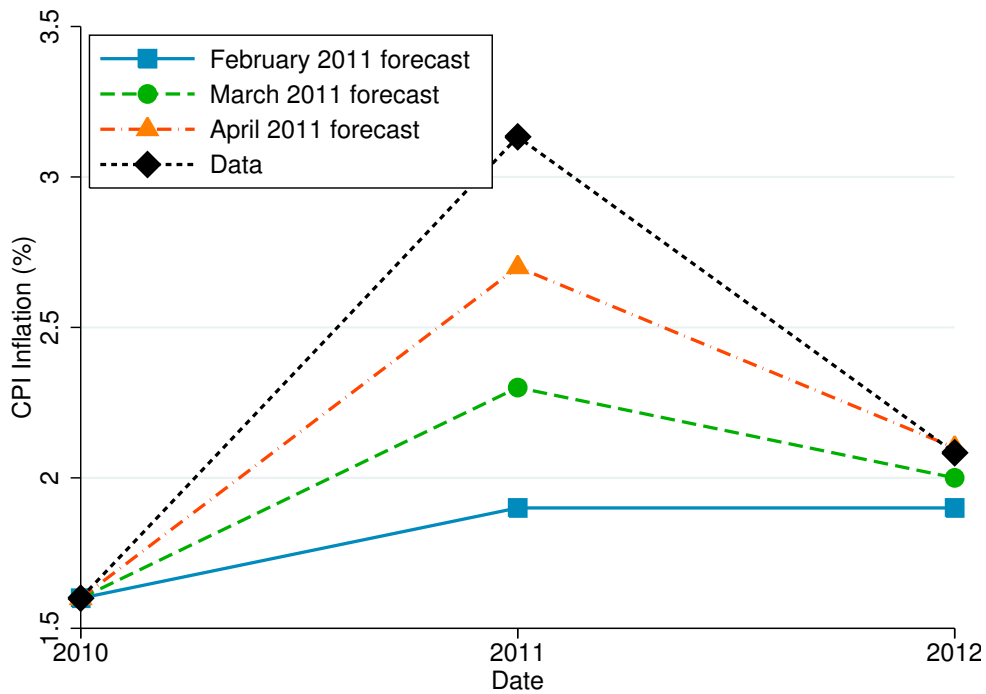
To determine the effects of this oil supply shock on expected inflation and real economic activity I proceed as in [section 3](#). The timing of the oil supply disruptions associated with the Libyan civil war are less precise than the timing of the Japanese earthquake. It could plausibly be dated on February 15th 2011 or to the beginning of foreign intervention on March 19th. Thus, I run an event study for the U.S. using both these dates. I omit Japan because the second event date occurs a few days after the Great East Japan Earthquake.

In [figure 8](#) I compare pre-February 15th forecasts for output and inflation with forecasts from March and April. The figure also display ex-post data. For both event dates expected inflation increases, consistent with the occurrence of a negative supply shock. In addition, for both event dates output forecasts were revised downwards and ex-post output came in even lower. These negative comovoments between expected inflation and output, as well

¹²I thank James Hamilton for suggesting this application.



(a) U.S.: GDP



(b) U.S.: CPI

Figure 8 – Consensus Economics forecasts from before the Libyan uprising (February 14th, 2011) and after (March 14th, 2011, and April 4th, 2011). Forecasts are for annual GDP and year-on-year inflation. GDP data is annual for 2010 and 2012 and quarterly from 2010Q4 until 2012Q1.

as actual inflation and output, suggest that the Libyan civil war and associated oil supply disruption were contractionary at the zero lower bound.

As before, I need to check that current and expected nominal interest rates do not rise following each event. For the March 19th event the 10-year nominal bond yield does rise by 6 basis points, but from February 14th to February 16th the 10-year nominal bond yield was unchanged. Thus, the outcomes for the February 14th event are inconsistent with the new Keynesian prediction, corroborating my results in [section 4](#).

5. Discussion

While my empirical analysis rejects the premise that negative supply shocks are expansionary at the zero lower bound, it does not point to a particular “fix” of the new Keynesian model. I therefore highlight several proposals in the literature that eliminate this predictions. These also overturn other unusual policy predictions for the zero lower bound, such as large fiscal multipliers.

[Cochrane \(2016\)](#) shows that the unusual policy predictions are a consequence of picking the equilibrium where inflation is exactly zero upon exiting the zero lower bound, $\pi(T) = 0$. This equilibrium explodes backward in time, generating a large fall in inflation and output when agents learn of a negative natural rate shock. However, any other equilibrium selection that bounds the initial jump in inflation, such as his backward-stable or the no-jump equilibrium, produces conventional outcomes: negative supply shocks are contractionary and fiscal multipliers are below 1 at the zero lower bound. For example, in the backward-stable equilibrium of the simple model in [section 2](#), the negative productivity shock lowers consumption and output under passive monetary policy,

$$y(t) = c(t) = \frac{\kappa\bar{a}}{\lambda_1 - \lambda_2} \left[\lambda_1(1 - e^{\lambda_2 t}) - \lambda_2(1 - e^{-\lambda_1(T-t)}) \right] < 0,$$

$$\pi(t) = \frac{\kappa\bar{a}}{\lambda_1 - \lambda_2} (e^{-\lambda_1(T-t)} - e^{\lambda_2 t}).$$

Inflation is initially positive as in the data, $\pi(0) > 0$, but gradually turns into deflation. Ultimately prices are lower in the long-run. Thus, expected real interest rates are high, inducing consumers today to postpone consumption, which generates the contraction.

[Braun et al. \(2016\)](#) solve a fully-nonlinear new Keynesian model, subject to a two-state Markov process where the low state corresponds to the zero lower bound. They highlight a bifurcation in the parameter space. When the persistence of the zero lower bound is moderate, then the model generates unusual policy predictions. But when the persistence of the zero lower bound is high, then labor tax cuts are expansionary and fiscal multipliers are below 1. Similarly, [Mertens and Ravn \(2014\)](#) obtain conventional comparative statics because their pessimism shock is highly persistent. Typically this part of the parameter space is ruled out by conventional new Keynesian equilibrium selection rules due to multiplicity. However, [Aruoba, Cuba-Borda, and Schorfheide \(2016\)](#) provide evidence that it is relevant for Japan. [Braun et al. \(2016\)](#) also show that labor tax cuts are expansionary and fiscal multipliers are small in the fully nonlinear model when the persistence of the zero lower bound is low.

[Kiley \(2016\)](#) replaces the sticky-price Phillips curve with a sticky-information Phillips curve in an otherwise standard new Keynesian model. This change anchors the long-run price level under passive monetary policy, which limits the scope for intertemporal substitution from negative supply shocks or fiscal policy. His model generates both a contraction from negative supply shocks and fiscal multipliers below 1 when the nominal interest rate is constant.

6. Conclusion

The standard new Keynesian model predicts that economies are governed by very different rules at the zero lower bound. In these models aggregate demand is determined by intertemporal substitution. Lower real interest rates induce consumers to pull consumption forward, which raises output. Lower productivity or higher government spending raises

marginal costs and expected inflation. If the central bank does not react to higher inflation, for example because it is constrained by the zero lower bound, then expected real interest rates fall causing a consumption and output boom.

I test this prediction and find it wanting. I study the Great East Japan Earthquake and oil supply shocks as two examples of negative supply shocks that raise the marginal cost of production. I verify that both raise expected inflation and do not raise expected nominal interest rates as required by theory, but I find that these events are still contractionary overall. I highlight variants of the new Keynesian model that are consistent with my empirical results. These also overturn other unusual policy prescriptions for the zero lower bound, such as large fiscal multipliers. This suggests that policy makers should be cautious in expecting large positive outcomes based on the standard new Keynesian model. Contrary to Krugman's claim, the zero lower bound world may not be so "topsy-turvy" after all.

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A. Solving the model

Following [Cochrane \(2016\)](#), I solve the model using lag operators. Differentiating equation (2) and combining with (1) yields a second-order difference equation in inflation,

$$\frac{d^2\pi(t)}{dt^2} - \rho \frac{d\pi(t)}{dt} - \kappa^* \pi(t) = -\kappa^* \left[i(t) - \rho - \frac{da(t)}{dt} \right].$$

A.1. Normal times

I substitute $i(t) = \rho + \phi\pi(t)$ to get,

$$\frac{d^2\pi(t)}{dt^2} - \rho \frac{d\pi(t)}{dt} + \kappa^*(\phi - 1)\pi(t) = \kappa^* \frac{da(t)}{dt}.$$

Using lag operators I factor the left-hand-side,

$$\left(\frac{d}{dt} - \mu_1 \right) \left(\frac{d}{dt} - \mu_2 \right) \pi(t) = \kappa^* \frac{da(t)}{dt},$$

where the eigenvalues μ_1, μ_2 are given in the text. The real part of both eigenvalues is positive. I next rewrite the factored equation as,

$$\pi(t) = \frac{1}{\mu_1 - \mu_2} \left[\frac{1}{\frac{d}{dt} - \mu_1} - \frac{1}{\frac{d}{dt} - \mu_2} \right] \kappa^* \frac{da(t)}{dt}.$$

The solution of the $\left(\frac{d}{dt} - \mu_1 \right) \pi(t) = \kappa^* \frac{da(t)}{dt}$ piece is,

$$\pi(t) = k_1 e^{\mu_1 t} - \kappa^* \int_{s=t}^{\infty} e^{-\mu_1(s-t)} \frac{da(s)}{ds} ds,$$

and the forward-boundedness assumption restricts $k_1 = 0$.

The solution of the $\left(\frac{d}{dt} - \mu_2 \right) \pi(t) = \kappa^* \frac{da(t)}{dt}$ piece is,

$$\pi(t) = k_2 e^{\mu_2 t} - \kappa^* \int_{s=t}^{\infty} e^{-\mu_2(s-t)} \frac{da(s)}{ds} ds,$$

and the forward-boundedness assumption restricts $k_2 = 0$.

Combining the two solutions I get

$$\pi(t) = \frac{\kappa^*}{\mu_1 - \mu_2} \int_{s=t}^{\infty} \left[-e^{-\mu_1(s-t)} + e^{-\mu_2(s-t)} \right] \frac{da(s)}{ds} ds.$$

Then use equation (2)

$$\kappa^* c(t) = \kappa^* a(t) + \rho \pi(t) - \frac{d\pi(t)}{dt}$$

to get

$$\kappa^* c(t) = \kappa^* a(t) - \frac{\kappa^*(\rho - \mu_1)}{\mu_1 - \mu_2} \int_{s=t}^{\infty} e^{-\mu_1(s-t)} \frac{da(s)}{ds} ds + \frac{\kappa^*(\rho - \mu_2)}{\mu_1 - \mu_2} \kappa^* \int_{s=t}^{\infty} e^{-\mu_2(s-t)} \frac{da(s)}{ds} ds.$$

Substitution of the step-function process for $a(t)$ yields the expression in the text. Note that under the assumed shock process the equilibrium also satisfies the $\pi(T) = 0$ restriction.

A.2. Zero Lower Bound

I substitute $i(t) = \rho$ to get,

$$\frac{d^2\pi(t)}{dt^2} - \rho \frac{d\pi(t)}{dt} - \kappa^* \pi(t) = \kappa^* \frac{da(t)}{dt}.$$

The factorization is now,

$$\left(\frac{d}{dt} - \lambda_1 \right) \left(\frac{d}{dt} - \lambda_2 \right) \pi(t) = \kappa^* \frac{da(t)}{dt},$$

where the eigenvalues $\lambda_1 > 0$ and $\lambda_2 < 0$ are given in the text.

Again rewrite this equation as

$$\pi(t) = \frac{1}{\lambda_1 - \lambda_2} \left[\frac{1}{\frac{d}{dt} - \lambda_1} - \frac{1}{\frac{d}{dt} - \lambda_2} \right] \kappa^* \frac{da(t)}{dt}.$$

The solution of the $\left(\frac{d}{dt} - \lambda_1 \right) \pi(t) = \kappa^* \frac{da(t)}{dt}$ piece is,

$$\pi(t) = k_1 e^{\lambda_1 t} - \kappa^* \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} \frac{da(s)}{ds} ds,$$

and the forward-boundedness assumption restricts $k_1 = 0$.

The solution of the $\left(\frac{d}{dt} - \lambda_2\right) \pi(t) = \kappa^* \frac{da(t)}{dt}$ piece is,

$$\pi(t) = k_2 e^{\lambda_2 t} + \kappa^* \int_{s=-\infty}^t e^{\lambda_2(t-s)} \frac{da(s)}{ds} ds,$$

which is stable for all k_2 since $\lambda_2 < 0$.

Combining the two solutions I get

$$\pi(t) = \frac{1}{\lambda_1 - \lambda_2} \left[-\kappa^* \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} \frac{da(s)}{ds} ds - k_2 e^{\lambda_2 t} - \kappa^* \int_{s=-\infty}^t e^{\lambda_2(t-s)} \frac{da(s)}{ds} ds \right].$$

To pin down k_2 I use the standard equilibrium selection $\pi(T) = 0$,

$$\pi(T) = \frac{1}{\lambda_1 - \lambda_2} \left[-\kappa^* \int_{s=T}^{\infty} e^{-\lambda_1(s-T)} \frac{da(s)}{ds} ds - k_2 e^{\lambda_2 T} - \kappa^* \int_{s=-\infty}^T e^{\lambda_2(T-s)} \frac{da(s)}{ds} ds \right] = 0.$$

(In [section 5](#) I instead select the backward-stable equilibrium $k_2 = 0$.)

Solving for k_2 and substituting into the expression for $\pi(t)$ for $0 \leq t \leq T$ yields,

$$\pi(t) = \frac{\kappa^*}{\lambda_1 - \lambda_2} \int_{s=t}^T \left[-e^{-\lambda_1(s-t)} + e^{\lambda_2(t-s)} \right] \frac{da(s)}{ds} ds.$$

Then use [equation \(2\)](#)

$$\kappa^* c(t) = \kappa^* a(t) + \rho \pi(t) - \frac{d\pi(t)}{dt}$$

to get

$$\kappa^* c(t) = \kappa^* a(t) - \frac{\kappa^*(\rho - \lambda_1)}{\lambda_1 - \lambda_2} \int_{s=t}^T e^{-\lambda_1(s-t)} \frac{da(s)}{ds} ds + \frac{\kappa^*(\rho - \lambda_2)}{\lambda_1 - \lambda_2} \int_{s=t}^T e^{\lambda_2(t-s)} \frac{da(s)}{ds} ds.$$

Again, substitution for the assumed path $a(t)$ yields the expression in the text.

B. Smets-Wouters Impulse Response Functions

I generate impulse response functions following [Bodenstein, Erceg, and Guerrieri \(2010\)](#) and [Coibion, Gorodnichenko, and Wieland \(2012\)](#). The [Smets and Wouters \(2007\)](#) model consists of a system of linearized difference equations,

$$A\mathbb{E}_t x_{t+1} + Bx_t + Cx_{t-1} + D\varepsilon_t = 0, \quad (7)$$

where x_t is a vector of endogenous variables, and ε_t is a vector of shocks. The locally unique forward-bounded solution around the zero-inflation steady state can be written as

$$x_t = Px_{t-1} + G\varepsilon_t.$$

Standard software packages such as Dyanare can be used to obtain the matrices P and G .

The impulse response function for normal times is then

$$x_t = P^t G \varepsilon_0.$$

To capture the zero lower bound regime, I use an interest rate peg that lasts for T quarters. Thus, I replace the interest rate rule in (7) with $i(t) = \rho$ for $0 \leq t \leq T$. Denote the new matrices A^* , B^* , C^* , and D^* . I solve the model under perfect foresight and impose the terminal condition $x_{T+1} = Px_T$. (The second step is equivalent to selecting the $\pi(T) = 0$ equilibrium in the simple model.) The impulse response function is,

$$x_0 = -(B^* + A^* M^{(0)})^{-1} D^* \varepsilon_0$$

$$x_t = M^{(t)} x_{t-1},$$

where the matrices $M^{(t)}$ are defined recursively,

$$M^{(T)} = -(B^* + A^* P)^{-1} C^*$$

$$M^{(t-1)} = -(B^* + A^* M^{(t)})^{-1} C^*.$$

C. Robustness

I document the robustness of my findings to alternative econometric specifications, samples, and VAR specifications.

First, I exclude lagged dependent variables from the regression in (6). If the oil supply shocks are well-identified then they should be orthogonal and the inclusion of lagged dependent variables is unnecessary. Further, equation (6) imposes an autoregressive structure on the dynamics, which may be at odds with the data. However, including lagged dependent variables does sharpen the precision of my estimates and guards against small-sample correlation of oil supply shocks with lagged outcomes. In the end, the impulse response functions with and without lagged dependent variables in [figure 9\(a\)](#) are quite similar.

Second, the VAR-identified oil supply shocks may have positive small sample correlation with the 2008 financial crisis. To address this concern, I set all oil supply shocks to zero from 2007 through 2009. [Figure 9\(b\)](#) shows that this robustness check only leads to minor changes in the impulse response function.

Third, rather than separating time-periods into a discrete zero lower bound regime and a normal-times regimes, I let the state vary continuously as in [Auerbach and Gorodnichenko \(2012\)](#) and [Ramey and Zubairy \(2014\)](#). I index the state by $F(z_t)$, where $0 \leq F(z) \leq 1$ captures the degree to which the zero lower bound binds. As indicator of the severity of the zero lower bound constraint, I let z_t be the centered seven-month moving average of the two-year bond yield. In the event that this value is negative, I set it to 0.01bp. Relative to shorter-maturity yields, the two-year bond yield also captures differences in the expected duration of the zero lower bound. I then define F as a logistic function following [Auerbach and Gorodnichenko \(2012\)](#),

$$F(z_t) = \frac{\exp\left\{-\theta \frac{\ln(z_t) - c}{\text{std}(\ln z_t)}\right\}}{1 + \exp\left\{-\theta \frac{\ln(z_t) - c}{\text{std}(\ln z_t)}\right\}},$$

where the parameter θ captures the smoothness between regimes. When $\theta = 0$ the regime is

constant, whereas if $\theta \rightarrow \infty$ the regime switches discretely between 0 and 1. The parameter c determines the switching point—when $\ln(z_t) = c$ then $F = 0.5$ for all finite θ . I set c such that the 75th percentile of z_t in my baseline zero lower bound regime corresponds to $F(z_t) = 0.5$. This is a conservative cut-off relative to my baseline delineation since for $\theta \rightarrow \infty$ it would drop 25% of my baseline zero lower bound sample. I let $\theta = 2$, which generates a smoother regime shift relative to the baseline.

I then estimate the local projection

$$\begin{aligned} \Delta y_{t+h} = & \alpha_h + \sum_{j=1}^m \beta_{j,h}^{F=0} [\Delta y_{t-j} \times (1 - F(z_t))] + \sum_{j=1}^m \beta_{j,h}^{F=1} (\Delta y_{t-j} \times F(z_t)) \\ & + \sum_{j=0}^k \gamma_{j,h}^{F=0} [oil_{t-j} \times (1 - F(z_t))] + \sum_{j=0}^k \gamma_{j,h}^{F=1} (oil_{t-j} \times F(z_t)) + \delta F(z_t) + \eta_{t+h} \end{aligned} \quad (8)$$

and display the coefficients $\{\gamma_{0,h}^{F=1}\}_{h=0}^{18}$ for the zero lower bound and $\{\gamma_{0,h}^{F=0}\}_{h=0}^{18}$ for normal times in [figure 10\(a\)](#). The impulse response functions are again similar to my baseline specification. I have also tried values of θ in the range from 1 to 10 and longer-maturity bond yields, but this did not notably change the results. This suggests that the two-state delineation in my baseline is a reasonable benchmark.

Fourth, I assess to what extent oil supply shocks are also contractionary in the U.S. when the zero lower bound is a binding constraint. The U.S. provides useful complementary evidence because it is also a major oil importer over the sample period, and it has now spent more than six years at the zero lower bound. A limitation of this that the inflation expectations response is not statistically significant¹³, and the confidence bands for expected nominal and real interest rates are quite wide. Nevertheless, the impulse response functions for unemployment in the U.S. ([Figure 10\(b\)](#)) is similar to Japan: there is a statistically significant increase in unemployment at the zero lower bound in the short-run, and this contraction is larger than in normal times.

Fifth, I estimate oil supply shocks from a regime-switching VAR. This helps assess if the

¹³For the U.S., I use the Michigan Survey inflation expectations data, which, unlike the Consensus Economics data, is available at monthly frequency.

linear VAR structure for the world oil market is a useful benchmark. I again index the state by a function $G(w_t)$, where $0 \leq G(w) \leq 1$ captures the degree to which the zero lower bound binds globally. I uncover oil supply shocks from the first equation of the regime-switching VAR,

$$\Delta prod_t = \alpha + \delta G(w_t) + \sum_{j=1}^{24} A_j^{G=0} [x_{t-j} \times (1 - G(w_t))] + \sum_{j=1}^{24} A_j^{G=1} [x_{t-j} \times G(w_t)] + \varepsilon_t.$$

where $x_t = (\Delta prod_t, rea_t, rpo_t)'$.

I use two different calibrations for the G function. First, I set it to 1 after January 2009. This parameterization captures that the zero lower bound was only a global phenomenon after the financial crisis.

Second, similar to the local projections approach, I let G be a logistic function

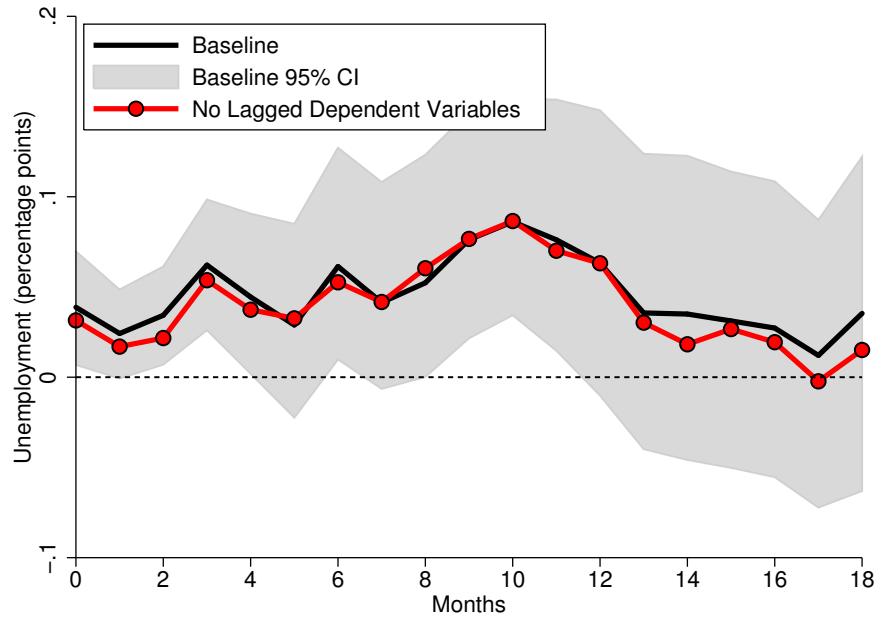
$$G(w_t) = \frac{\exp\left\{-\theta \frac{\ln(w_t) - c}{std(\ln w_t)}\right\}}{1 + \exp\left\{-\theta \frac{\ln(w_t) - c}{std(\ln w_t)}\right\}},$$

where w_t is an oil consumption weighted average of national overnight nominal interest rates. I construct this variable using the overnight nominal interest rate data from the OECD, which includes all OECD members, the BRICS, Indonesia and Lithuania. I weight them using oil consumption data from the EIA, the same source as the oil production data. These countries account for over 70% of global oil consumption in the 2000s. I then take the seven-month centered moving average of this global interest rate.

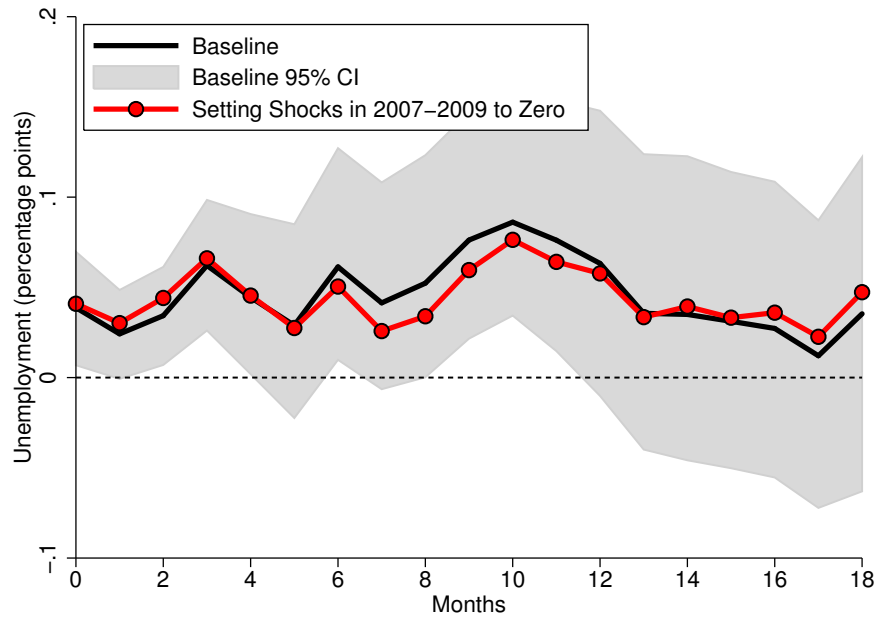
I chose a calibration of c and θ to allow low interest rates to affect the dynamics of the VAR, even though the average interest rate w_t never falls below 2% in my sample, which suggests a more limited role for the zero lower bound. I set $c = \ln(3)$, since w_t falls below 3% for the first time to in February 2009. This aligns with the discrete regime, which switches to 1 in January 2009. I again set $\theta = 2$, which yields relatively smooth variation for the zero lower bound state compared to the discrete-regime VAR. It implies that a world with an average interest rate of 2.9% is not that different from a world with an average interest rate of 3.1%. This is a useful counterpoint to the discrete-regime, where these two worlds are

in different regimes. (Letting $\theta \rightarrow \infty$ makes the smooth-regime very close to the discrete-regime, with the only difference that the former becomes 1 in February 2009 and the latter in January 2009.)

Using these oil supply shock series, I construct impulse response functions for Japan as before. **Figure 11** shows that the results are very similar to the baseline. Further, there is relatively little difference between the discrete-regime and the smooth-regime VAR.

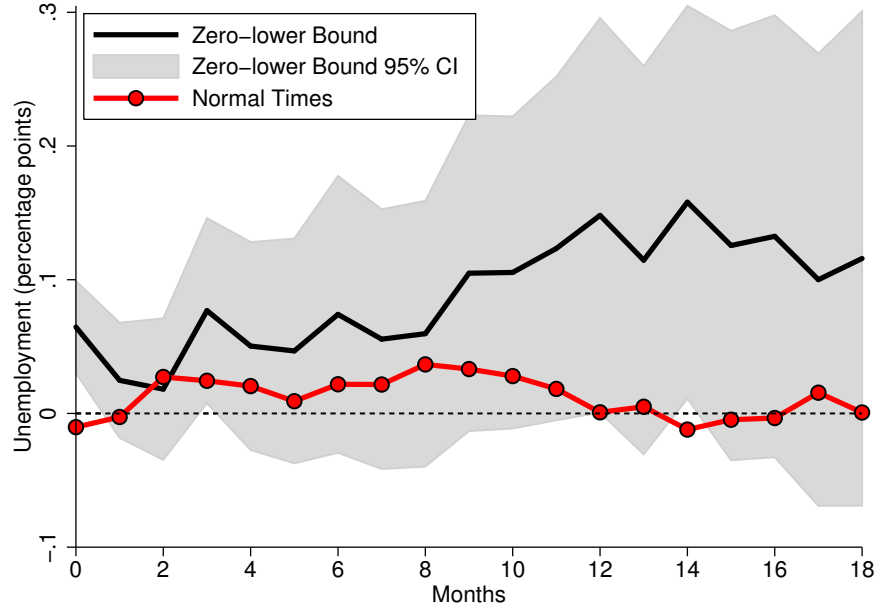


(a) No lagged dependent variables

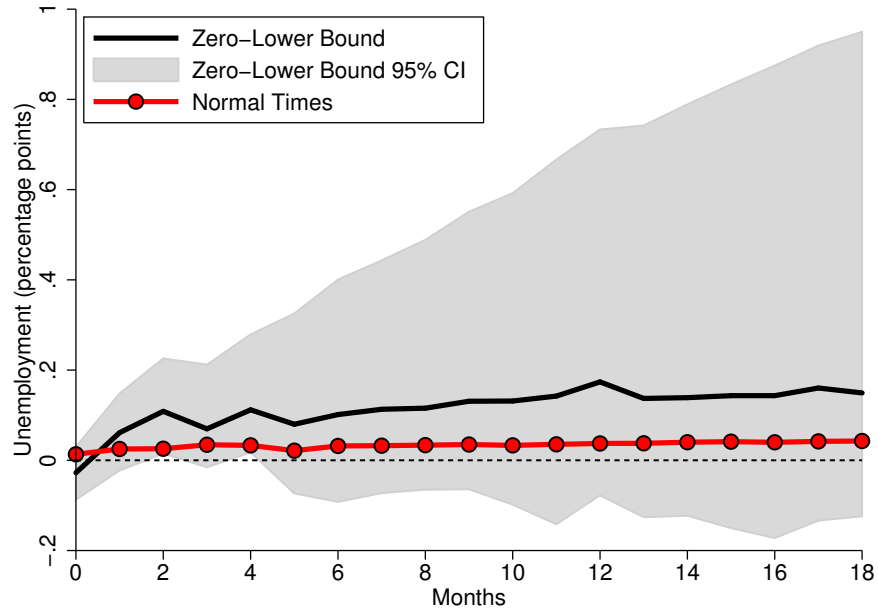


(b) Setting oil supply shocks to zero from 2007-2009

Figure 9 – Robustness checks on impulse response functions of unemployment to negative oil supply shocks. These are constructed from estimates of autoregressive distributive lag equation (6) in changes or growth rates and aggregated to levels. 95% confidence intervals are derived from a parametric bootstrap based on a heteroscedasticity and autocorrelation-robust covariance matrix and corrected for the presence of estimated regressors following [Murphy and Topel \(2002\)](#). Sample and lag lengths are as in the baseline.

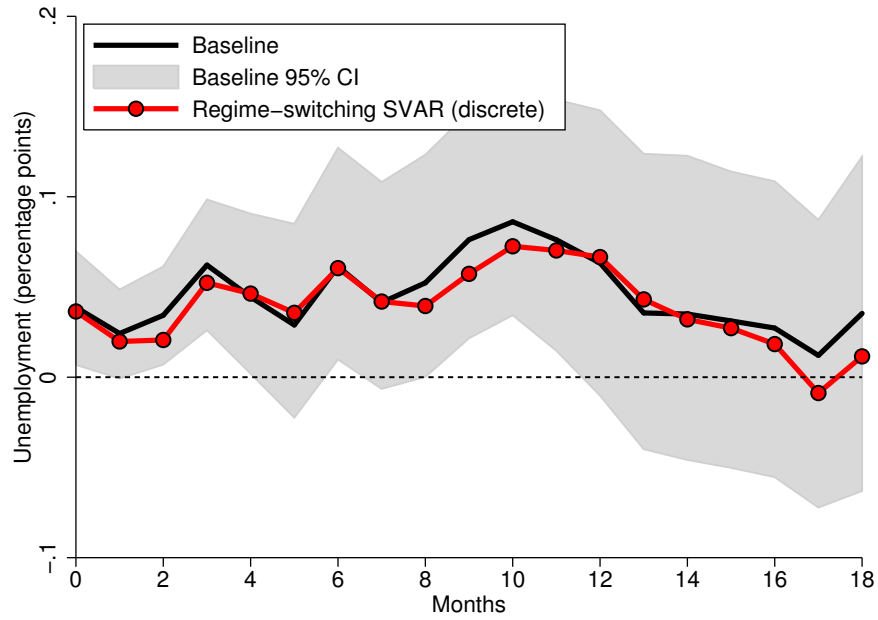


(a) Local projections approach

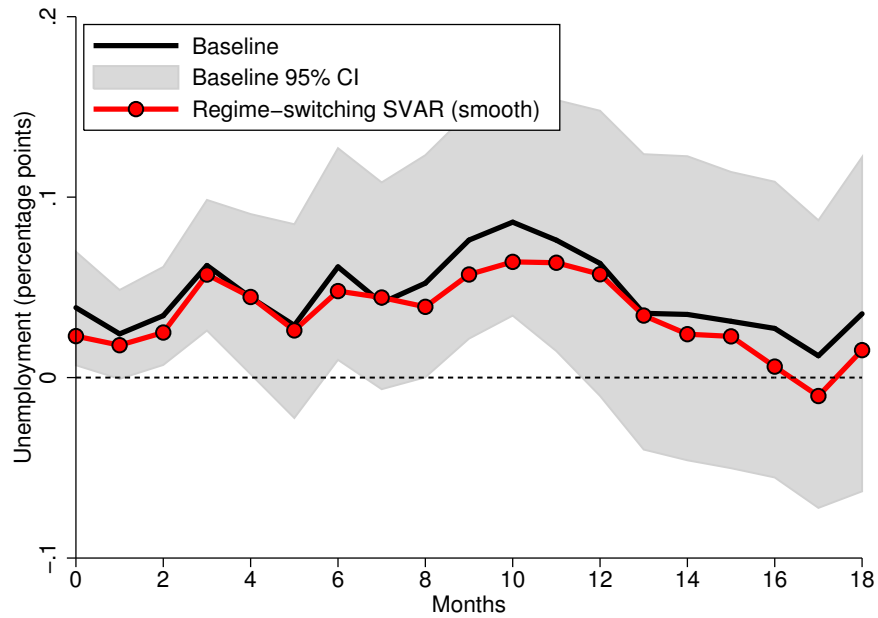


(b) U.S. Unemployment

Figure 10 – Robustness checks on impulse response functions of unemployment to negative oil supply shocks. These are constructed from estimates of autoregressive distributive lag equation (6) or local projection equation (8) in changes or growth rates and aggregated to levels. 95% confidence intervals are derived from a parametric bootstrap based on a heteroscedasticity and autocorrelation-robust covariance matrix and corrected for the presence of estimated regressors following [Murphy and Topel \(2002\)](#). Sample and lag lengths are as in the baseline for figure (a). For the U.S., lag lengths are $m = 12$ and $k = 6$ and the zero lower bound regime begins in 1/2009.



(a) Discrete-regime VAR



(b) Smooth-regime VAR

Figure 11 – Robustness checks on impulse response functions of unemployment to negative oil supply shocks. These are constructed from estimates of autoregressive distributive lag equation (6) in changes or growth rates and aggregated to levels. 95% confidence intervals are derived from a parametric bootstrap based on a heteroscedasticity and autocorrelation-robust covariance matrix and corrected for the presence of estimated regressors following [Murphy and Topel \(2002\)](#). Sample and lag lengths are as in the baseline.

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D. Continuous-time new Keynesian Phillips curve

A continuum of firms indexed by $i \in [0, 1]$ produces output Y_i with the technology

$$Y_i(t) = A(t)L_i(t).$$

where A is aggregate productivity and L_i is labor input. I assume that labor is perfectly mobile across firms.

Relative demands for each firm are,

$$Y_i(t) = \left(\frac{P_i(t)}{P(t)} \right)^{-\theta} Y(t).$$

where P_i is the nominal price of variety i , P is the aggregate price index and $\theta > 1$ is the elasticity of substitution across goods. Aggregate output is defined as,

$$Y(t) = \left(\int_0^1 Y_i(t)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}.$$

and aggregate price index is,

$$P(t) = \left(\int_0^1 P_i(t)^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$

The gross inflation rate of the aggregate price index is denoted by $\Pi(t)$.

With Poisson intensity λdt a firm can reset its nominal price. The optimal reset price $P_i^*(t)$ maximizes the sum of discounted profits while the price is not reset,

$$\max_{P_i^*(t)} \int_t^\infty e^{-(\rho+\lambda)(s-t)} C(s)^{-\sigma} \left[(1 + \tau) \frac{P_i^*(t)}{P(s)} Y_i(s) - \frac{W(s)}{P(s)} L_i(s) \right] ds,$$

where ρ is the discount rate, $C^{-\sigma}$ captures the contingent valuation of real profits, τ is an output subsidy and $\frac{W}{P}$ is the real wage. Because labor is perfectly mobile, the latter is the same across firms.

Substituting for relative demand and the production function yields,

$$\max_{P_i^*(t)} \int_t^\infty e^{-(\rho+\lambda)(s-t)} C(s)^{-\sigma} \left[(1+\tau) \left(\frac{P_i^*(t)}{P(s)} \right)^{1-\theta} Y(s) - \frac{W(s)}{P(s)} \left(\frac{P_i^*(t)}{P(s)} \right)^{-\theta} \left(\frac{Y(s)}{A(s)} \right) \right] ds.$$

The first order condition for the firms real reset price is,

$$\left(\frac{P_i^*(t)}{P(t)} \right) = \frac{\theta}{(\theta-1)(1+\tau)} \frac{\int_t^\infty e^{-(\rho+\lambda)(s-t)} C(s)^{-\sigma} \frac{W(s)}{P(s)} \left(\frac{P(t)}{P(s)} \right)^{-\theta} \left(\frac{Y(s)}{A(s)} \right) ds}{\int_t^\infty e^{-(\rho+\lambda)(s-t)} C(s)^{-\sigma} \left(\frac{P(t)}{P(s)} \right)^{1-\theta} Y(s) ds}.$$

Since the right-hand-side is independent of i , each firm would pick the same reset price at t . I assume that the output subsidy is set such that the steady state mark-up is zero, $\tau = \frac{\theta}{\theta-1} - 1$.

The log-linear approximation around the zero-inflation steady state is,

$$b(t) = (\rho + \lambda) \int_t^\infty e^{-(\rho+\lambda)(s-t)} \left[\omega(s) - a(s) + \frac{1}{\rho + \lambda} \pi(s) \right] ds$$

where $b(t) = \log\left(\frac{P^*(t)}{P(t)}\right)$, $\omega(t) = \log\left(\frac{W(t)}{P(t)}\right) - \log\left(\frac{\bar{W}}{\bar{P}}\right)$, $a(t) = \log(a(t)) - \log(\bar{a})$, $y(t) = \log(y(t)) - \log(\bar{y})$ and $\pi(t) = \log(\Pi(t))$.

We can rewrite this first order condition as a differential equation,

$$db(t) = -(\rho + \lambda) \left[\omega(t) - a(t) + \frac{1}{\rho + \lambda} \left(1 + \frac{\alpha\theta}{1-\alpha} \right) \pi(t) \right] dt + (\rho + \lambda)b(t)dt$$

Without indexation, the gross inflation rate is solely a function of the reset price and the Calvo intensity,

$$\pi(t) = \frac{\lambda}{1-\theta} \left[\left(\frac{P^*(t)}{P(t)} \right)^{1-\theta} - 1 \right]$$

The log-linear approximation to this law of motion is,

$$\pi(t) = \lambda b(t),$$

which implies,

$$d\pi(t) = \lambda db(t).$$

Combining these two expressions for $b(t)$ and $db(t)$ with the firm's first order condition yields,

$$d\pi(t) = -\kappa[\omega(t) - a(t)] dt + \rho\pi(t)dt$$

where $\kappa = \lambda(\rho + \lambda)$.

Equation (2) obtains with period utility function $\ln C(t) - \chi L(t)$,

$$\omega(t) = c(t) = y(t)$$

$$\kappa^* = \kappa$$

where the first equation is the first order condition for household labor supply.

The new Keynesian Phillips curve with government spending obtains with period utility function $\ln C(t) - \chi \frac{L(t)^{1+\nu}}{1+\nu}$ and a steady state share of government spending $s_g > 0$,

$$\omega(t) = c(t) + \nu l(t) = (1 + \nu(1 - s_g))c(t) + \nu s_g g(t) - \nu a(t)$$

$$\kappa^* = \kappa(1 + \nu(1 - s_g))$$

$$\psi_a = 1 + \nu$$

$$\psi_g = \nu s_g$$

E. Model with explicit zero lower bound

This section shows that an interest rate peg yields the same outcome for the negative productivity shock as a scenario where the zero lower bound binds until T .

I now allow for a shock to the natural rate of interest $r(T)$ as in [Werning \(2012\)](#) and explicitly incorporate the zero lower bound in the interest rate rule,

$$\begin{aligned}\frac{dc(t)}{dt} &= i(t) - \pi(t) - r(t) \\ \frac{d\pi(t)}{dt} &= \rho\pi(t) - \kappa^*[c(t) - a(t)] \\ i(t) &= \max\{\rho + \phi\pi(t), 0\}, \quad \phi > 1\end{aligned}$$

As before there is a productivity supply shock until time T ,

$$\begin{aligned}a(t) &= \bar{a} < 0 & 0 \leq t < T, \\ a(t) &= 0 & t \geq T.\end{aligned}$$

I also assume that the natural rate of interest is sufficiently negative that the zero lower bound binds until time T ,¹⁴

$$\begin{aligned}r(t) &= \bar{r} < 0 & 0 \leq t < T, \\ r(t) &= \rho & t \geq T.\end{aligned}$$

After T , the central bank implements $\pi(T) = 0$ per the usual equilibrium selection. This immediately implies that $c(t) = \pi(t) = 0$ for $t \geq T$.

¹⁴A sufficient condition is

$$\frac{\kappa^*}{\lambda_1 - \lambda_2} \left[\frac{1}{\lambda_2} (1 - e^{-\lambda_2(T-t)}) - \frac{1}{\lambda_1} (1 - e^{-\lambda_1(T-t)}) \right] \bar{r} + \frac{\kappa^*}{\lambda_1 - \lambda_2} (e^{-\lambda_1(T-t)} - e^{-\lambda_2(T-t)}) \bar{a} < -\rho$$

The model dynamics for $0 \leq t < T$ are,

$$\frac{dc(t)}{dt} = -\pi(t) - \bar{r} \quad (9)$$

$$\frac{d\pi(t)}{dt} = \rho\pi(t) - \kappa^*[c(t) - \bar{a}] \quad (10)$$

This model is linear with boundary condition $\pi(T) = 0$, which can be solved using standard methods. The solution is,

$$\begin{aligned} c(t) &= \frac{1}{\lambda_1 - \lambda_2} \left[\frac{\lambda_1}{\lambda_2} (1 - e^{-\lambda_2(T-t)}) - \frac{\lambda_2}{\lambda_1} (1 - e^{-\lambda_1(T-t)}) \right] \bar{r} \\ &\quad + \frac{1}{\lambda_1 - \lambda_2} \left[\lambda_1 (1 - e^{-\lambda_2(T-t)}) - \lambda_2 (1 - e^{-\lambda_1(T-t)}) \right] \bar{a} \\ \pi(t) &= \frac{\kappa^*}{\lambda_1 - \lambda_2} \left[\frac{1}{\lambda_2} (1 - e^{-\lambda_2(T-t)}) - \frac{1}{\lambda_1} (1 - e^{-\lambda_1(T-t)}) \right] \bar{r} + \frac{\kappa^*}{\lambda_1 - \lambda_2} (e^{-\lambda_1(T-t)} - e^{-\lambda_2(T-t)}) \bar{a} \end{aligned}$$

The coefficients on \bar{a} are identical to those reported in the text, as was to be shown.

F. A small-open-economy model with oil imports

I consider the case of a small open economy that imports oil for the purpose of production. It pays for these imports by exporting the produced output. Home agents maximize the stream of utility,

$$\int_0^\infty e^{-\rho t} \left[\frac{C(t)^{1-\sigma}}{1-\sigma} - \chi L(t) \right] dt,$$

where $C(t)$ is domestic consumption and $L(t)$ is labor. The inverse of the intertemporal elasticity of substitution is σ and χ is a parameter that determines steady state labor supply.

Domestic consumption $C(t)$ is an aggregate of a produced good C^y and consumed oil O^c ,

$$C(t) = \left[(1-\gamma)^{\frac{1}{\zeta}} C^y(t)^{\frac{\zeta-1}{\zeta}} + \gamma^{\frac{1}{\zeta}} O^c(t)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}.$$

The produced good is a combination of individual varieties,

$$C^y(t) = \left(\int_0^1 C_i(t)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Home asset holdings of the risk-free bond $D(t)$ evolve according to

$$dD(t) = [i(t)D(t) - P(t)C(t) + W(t)L(t) + \Pi(t)]dt,$$

where $i(t)$ is the nominal interest rate, $W(t)$ the wage rate, and $\Pi(t)$ are profits from firms.

Firms produce output $Y_i(t)$ of variety i according to a CES technology,

$$Y_i(t) = \left[(1-\xi)^{\frac{1}{\psi}} [A(t)L_i(t)]^{\frac{\psi-1}{\psi}} + \xi^{\frac{1}{\psi}} O_i^y(t)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}$$

where $O_i(t)$ is oil input, ξ the share of oil in production, and ψ its elasticity of substitution with labor input. Firms face standard Calvo pricing frictions.

Total imports of oil by the small home economy are

$$O(t) = \int_0^1 O_i^y(t) di + O^c(t)$$

The foreign economy is large relative to the domestic economy. It exports oil to the home

economy in exchange for produced goods $C^{yH^*}(t)$, where H^* denotes an import of the home good by the foreign country. Its consumption bundle is given by

$$C^*(t) = \left[(1 - \gamma)^{\frac{1}{\zeta}} C^{y^*}(t)^{\frac{\zeta-1}{\zeta}} + \gamma^{\frac{1}{\zeta}} O^{c^*}(t)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}.$$

which is analogous to the home country. I denote foreign quantities with a *, i.e. $C^*(t)$ is foreign consumption.

Foreign consumption of produced goods is a combination of imported goods and locally-produced goods,

$$C^{y^*}(t) = \left[(1 - \alpha)^{\frac{1}{\eta}} (C^{yF^*}(t))^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C^{yH^*}(t))^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where α is the share of foreign goods in consumption produced consumption and η the elasticity of substitution among home and foreign goods. Because the foreign economy is large, the share of home goods in foreign consumption is close to zero, $\alpha^* \rightarrow 0$.

The foreign economy is endowed with an exogenous supply of oil $O^*(t)$ and the real price of oil in foreign goods $\frac{P^{o^*}(t)}{P^*(t)}$ will adjust such that the oil market clears,

$$O^*(t) = \int_0^1 O_i^{y^*}(t) di + O^{c^*}(t)$$

Note that because the home economy is small, it has no influence on the oil market and thus no influence on the foreign real oil price. In all other aspects the foreign economy has identical preferences and constraints.

I assume that the law of one price holds. I then define the following relative prices. The real exchange rate $Q(t)$ is equal to

$$Q(t) = \frac{\mathcal{E}(t)P^*(t)}{P(t)},$$

where $\mathcal{E}(t)$ is the nominal exchange rate. The terms of trade,

$$S(t) = \frac{P^o(t)}{P^y(t)},$$

is equal to the ratio of the domestic oil price P^o and the consumption good price P^y .

The net foreign asset position of the home economy (denominated in home currency) evolves according to

$$dNFA(t) = [P^y(t)C^{yH^*}(t) - P^o(t)O(t)]dt + i(t)NFA(t)dt,$$

F.1. Log-linearization

The log-linearized equations of the domestic economy are

$$dc(t) = \sigma^{-1}[i(t) - \rho - \pi(t)]dt$$

$$\pi(t) = \pi^y(t) + \gamma dp^o(t)/dt$$

$$d\pi^y(t) = \rho\pi(t)^y dt - \kappa mc(t)dt$$

$$i(t) = \max\{r(t) + \phi_\pi\pi(t), 0\}, \quad \phi_\pi > 1$$

$$\omega(t) = \sigma c(t)$$

$$y(t) = (1 - \xi)a(t) + (1 - \xi)l(t) + \xi o^y(t)$$

$$y(t) = (1 - \xi)(1 - \gamma)c^y(t) + [1 - (1 - \xi)(1 - \gamma)]c^{yH^*}(t)$$

$$c^y(t) = o^c(t) + \frac{\zeta}{1 - \gamma}p^o(t)$$

$$c(t) = (1 - \gamma)c^y(t) + \gamma o^c(t)$$

$$mc(t) = \omega(t) - \psi^{-1}(y(t) - l(t)) - (1 - \psi^{-1})a(t)$$

$$o^y(t) = l(t) - \psi(p^o(t) - \omega(t)) - (\psi - 1)a(t)$$

$$o(t) = \frac{\gamma}{(1 - \gamma)\xi + \gamma}o^c(t) + \frac{(1 - \gamma)\xi}{(1 - \gamma)\xi + \gamma}o^y(t)$$

$$p^o(t) = p^{o*}(t) + q(t)$$

$$c^{yH^*}(t) = c^*(t) + \eta \frac{1}{1 - \gamma}q(t) + \zeta \frac{\gamma}{1 - \gamma}p^{o*}(t)$$

where lower-case letters denote log-deviations from steady state. p^o is the (domestic) real price of oil in terms of the produced good. An analogous set of equations governs the foreign

economy, with the exception that $y^*(t) = c^{y^*}(t)$ since that economy only exports oil,

$$\begin{aligned}
dc^*(t) &= \sigma^{-1}[i^*(t) - \rho - \pi^*(t)]dt \\
\pi^*(t) &= \pi^{y^*}(t) + \gamma dp^{o^*}(t)/dt \\
d\pi^{y^*}(t) &= \rho\pi^{y^*}(t)dt - \kappa mc^*(t)dt \\
i^*(t) &= \max\{r(t) + \phi_\pi \pi^*(t), 0\}, \quad \phi_\pi > 1 \\
\omega^*(t) &= \sigma c^*(t) \\
y^*(t) &= (1 - \xi)a(t) + (1 - \xi)l^*(t) + \xi o^{y^*}(t) \\
y^*(t) &= c^{y^*}(t) \\
c^{y^*}(t) &= o^{c^*}(t) + \frac{\zeta}{1 - \gamma} p^{o^*}(t) \\
c^*(t) &= (1 - \gamma)c^{y^*}(t) + \gamma o^{c^*}(t) \\
mc^*(t) &= \omega^*(t) - \psi^{-1}(y^*(t) - l^*(t)) - (1 - \psi^{-1})a(t) \\
o^{y^*}(t) &= l^*(t) - \psi(p^{o^*}(t) - \omega^*(t)) - (\psi - 1)a(t) \\
o^*(t) &= \frac{\gamma}{(1 - \gamma)\xi + \gamma} o^{c^*}(t) + \frac{(1 - \gamma)\xi}{(1 - \gamma)\xi + \gamma} o^{y^*}(t)
\end{aligned}$$

The log-linearized equations of the domestic economy can be reduced to

$$\begin{aligned}
dc(t) &= \sigma^{-1}[i(t) - \rho - \pi(t)]dt \\
\pi(t) &= \pi^y(t) + \gamma[dp^{o^*}(t) + dq(t)]/dt \\
d\pi^y(t) &= \rho\pi^y(t)dt - \kappa\{(1 - \xi)\sigma c(t) - (1 - \xi)a(t) + \xi[p^{o^*}(t) + q(t)]\}dt
\end{aligned}$$

while the foreign economy is rewritten as

$$\begin{aligned}
dc^*(t) &= \sigma^{-1}[i^*(t) - \rho - \pi^*(t)]dt \\
\pi^*(t) &= \pi^{y^*}(t) + \gamma dp^{o^*}(t)/dt \\
d\pi^{y^*}(t) &= \rho\pi^{y^*}(t)dt - \kappa\{(1 - \xi)\sigma c^*(t) - (1 - \xi)a(t) + \xi p^{o^*}(t)\}dt
\end{aligned} \tag{11}$$

The real price of oil is determined by market clearing in the foreign economy,

$$p^{o*}(t) = Q_c c^*(t) - Q_o o^*(t) - Q_a a(t) \quad (12)$$

where the constants Q_c, Q_o, Q_a are positive,

$$\begin{aligned} Q_c &= \frac{\gamma + (1 - \gamma)\xi + \sigma\psi\xi(1 - \gamma)(1 - \xi)}{(1 - \xi)[\gamma\zeta + (1 - \gamma)\xi\psi]} \\ Q_o &= \frac{\gamma + (1 - \gamma)\xi}{(1 - \xi)[\gamma\zeta + (1 - \gamma)\xi\psi]} \\ Q_a &= \frac{(1 - \gamma)\psi\xi}{\gamma\zeta + (1 - \gamma)\xi\psi} \end{aligned}$$

The negative supply shock is a temporary disturbance to world oil supply,

$$\begin{aligned} o^*(t) &= \bar{o} < 0 & 0 \leq t < T, \\ o^*(t) &= 0 & t \geq T. \end{aligned}$$

To solve for the domestic economy's allocation, I also need to specify the degree of market (in)completeness.

F.2. Case 1: complete international financial markets

When financial markets are complete, domestic and foreign consumption are related by the Backus-Smith condition

$$C(t) = \Theta C^*(t) Q(t)^{\frac{1}{\sigma}},$$

where Θ is the relative Pareto weight. The log-linearized equation is

$$c(t) = c^*(t) + \sigma^{-1} q(t)$$

I report solutions for two cases. First, when both economies follow an active interest rate

policy,

$$i(t) = \rho + \phi\pi(t),$$

$$i^*(t) = \rho + \phi\pi^*(t),$$

where $\phi > 1$ and second, when both economies follow an interest rate peg

$$i(t) = \rho$$

$$i^*(t) = \rho$$

Define the following parameters:

$$\begin{aligned} M^{NT} &= \kappa[(1 - \xi)\sigma + \xi Q_c] \frac{\sigma^{-1}(\phi - 1)}{1 - \gamma Q_c} > 0 \\ K^{NT} &= \kappa \left[\xi Q_o + \{(1 - \xi)\sigma + \xi Q_c\} \frac{\sigma^{-1}(\phi - 1)\gamma Q_o}{1 - \sigma^{-1}(\phi - 1)\gamma Q_c} \right] \\ M^{ZLB} &= -\kappa[(1 - \xi)\sigma + \xi Q_c] \frac{\sigma^{-1}}{1 - \gamma Q_c} < 0 \\ K^{ZLB} &= \kappa \left[\xi Q_o - \{(1 - \xi)\sigma + \xi Q_c\} \frac{\sigma^{-1}\gamma Q_o}{1 + \sigma^{-1}\gamma Q_c} \right] \end{aligned}$$

The solution for active monetary policy is

$$\begin{aligned} \kappa c(t) = \kappa c^*(t) &= \frac{1}{\tilde{\mu}_1 - \tilde{\mu}_2} \left[\tilde{\mu}_1(1 - e^{\tilde{\mu}_2(T-t)}) - \tilde{\mu}_2(1 - e^{-\tilde{\mu}_1(T-t)}) \right] K^{NT} \bar{o} \\ \pi^y(t) = \pi^{y*}(t) &= \frac{1}{\tilde{\mu}_1 - \tilde{\mu}_2} (e^{-\tilde{\mu}_1(T-t)} - e^{-\tilde{\mu}_2(T-t)}) K^{NT} \bar{o} \end{aligned}$$

where the eigenvalues are,

$$\tilde{\mu}_1 = \frac{\rho}{2} + \frac{\sqrt{\rho^2 - 4M^{NT}}}{2}, \quad \tilde{\mu}_2 = \frac{\rho}{2} - \frac{\sqrt{\rho^2 - 4M^{NT}}}{2}.$$

The solution for the constant interest rate rule is

$$\begin{aligned} \kappa c(t) = \kappa c^*(t) &= \frac{1}{\tilde{\lambda}_1 - \tilde{\lambda}_2} \left[\tilde{\lambda}_1(1 - e^{\tilde{\lambda}_2(T-t)}) - \tilde{\lambda}_2(1 - e^{-\tilde{\lambda}_1(T-t)}) \right] K^{ZLB} \bar{o} \\ \pi^y(t) = \pi^{y*}(t) &= \frac{1}{\tilde{\lambda}_1 - \tilde{\lambda}_2} (e^{-\tilde{\lambda}_1(T-t)} - e^{-\tilde{\lambda}_2(T-t)}) K^{ZLB} \bar{o} \end{aligned}$$

where the eigenvalues are,

$$\tilde{\lambda}_1 = \frac{\rho}{2} + \frac{\sqrt{\rho^2 - 4M^{ZLB}}}{2} > 0, \quad \tilde{\lambda}_2 = \frac{\rho}{2} - \frac{\sqrt{\rho^2 - 4M^{ZLB}}}{2} < 0.$$

If inflation rises at the zero lower bound given $\bar{o} < 0$, it must be that $K^{ZLB} > 0$. It then immediately follows that the negative supply shock raises consumption, $c(t) = c^*(t) > 0$. GDP in the model is equivalent to labor input $l(t)$, which also expands

$$l(t) = \underbrace{\left(1 - \sigma\xi\psi + \xi\psi Q_c + \frac{\gamma\zeta}{1-\gamma} Q_c\right)}_{>0} c(t) - \left(\xi\psi + \frac{\gamma\zeta}{1-\gamma}\right) Q_o \bar{o} > 0$$

This verifies the claim in the text.

F.3. Case 2: incomplete international financial markets

For this case, I assume that the only asset traded internationally is a one-period bond. While the equilibrium under complete markets featured $\Theta = 1$, in the equilibrium under incomplete markets $NFA(0)$ is given. [Farhi and Werning \(2012\)](#) show that the incomplete market allocation is the sum of two components – the complete market allocation, denoted CM , and an additional term, denoted δ_x^{IM} for variable x ,

$$\begin{aligned} c^{IM}(t) &= c^{CM}(t) + \delta_c^{IM}, & \pi^{IM}(t) &= \pi^{CM}(t) + \delta_\pi^{IM}, & o^{c,IM}(t) &= o^{c,CM}(t) + \delta_{o^c}^{IM}, \\ o^{y,IM}(t) &= o^{y,CM}(t) + \delta_{o^y}^{IM}, & y^{IM}(t) &= y^{CM}(t) + \delta_y^{IM}, & l^{IM}(t) &= l^{CM}(t) + \delta_l^{IM}, \\ q^{IM}(t) &= q^{CM}(t) + \delta_q^{IM}. \end{aligned}$$

I solve for the incomplete market terms using the relationship between foreign and home consumption,

$$c(t) = \theta + c^*(t) + \frac{1}{\sigma} q(t).$$

where $\theta = \ln \Theta$ is new (post oil supply shock) Pareto weight. One can interpret $\theta < 0$ as a wealth transfer to the foreign economy. Given θ , we can calculate the incomplete market

component of the home allocation as follows

$$\begin{aligned}
\delta_c^{IM} &= \xi\theta, & \delta_y^{IM} &= -(1-\xi)\{\xi(\sigma\eta-1) + \gamma\xi + \gamma(1-\xi)\sigma\zeta + \frac{\gamma}{1-\gamma}\sigma\eta\}\theta, \\
\delta_q^{IM} &= -(1-\xi)\sigma\theta, & \delta_l^{IM} &= \delta_y^{IM} - \sigma\xi\psi\theta, \\
\delta_\pi^{IM} &= 0, & \delta_{oy}^{IM} &= \delta_y^{IM} + \sigma(1-\xi)\psi\theta, \\
&& \delta_{oc}^{IM} &= [\xi + (1-\xi)\sigma\zeta]\theta,
\end{aligned}$$

Thus, the δ^{IM} -terms are constant because the home economy is forward looking and thus instantaneously adjusts to the new wealth level.

The value for θ is determined by the balanced-trade condition. Define $N\tilde{F}A(t) = \frac{C(s)^{-\sigma}}{P(s)} NFA(t)$ as real financial assets in utility units. I let zero net financial assets, $NFA(0) = 0$, be the initial condition. Given the no-Ponzi scheme condition, $N\tilde{F}A(t)$ must satisfy,

$$N\tilde{F}A(t) = \int_t^\infty \frac{C(s)^{-\sigma}}{P(s)} \left(C^{yH^*}(s) - \frac{P(s)}{Py(s)} \frac{P^o(s)}{P(s)} O(s) \right) ds.$$

Log-linearizing this condition and using $N\tilde{F}A(0) = 0$ we obtain,

$$\int_0^\infty e^{-\rho s} c^{yH^*}(s) ds = \int_0^\infty e^{-\rho s} \left[\frac{1}{1-\gamma} p^o(s) + o(s) \right] ds,$$

which states that trade must be balanced in the long-run.

Substituting the linearized equation into the international budget constraint yields a solution for the wealth effect θ ,

$$\theta = \frac{1}{\sigma(\eta-1)(1-\xi)(1-\gamma)^{-1} + \frac{\gamma}{(1-\gamma)\xi+\gamma} \frac{\delta_{oc}^{IM}}{\theta} + \frac{(1-\gamma)\xi}{(1-\gamma)\xi+\gamma} \frac{\delta_{oy}^{IM}}{\theta}} \int_0^T [c^{*,CM}(s) - \frac{1}{1-\gamma} p^{o*,CM}(s) - o^{*,CM}(s)] ds$$

Thus consumption, gross output and employment (GDP) are equal to,

$$\begin{aligned}
c^{IM}(t) &= c^{CM}(t) + \xi\theta \\
y^{IM}(t) &= y^{CM}(t) - (1-\xi)\{\xi(\sigma\eta-1) + \gamma\xi + \gamma(1-\xi)\sigma\zeta + \frac{\gamma}{1-\gamma}\sigma\eta\}\theta \\
l^{IM}(t) &= l^{CM}(t) - (1-\xi)\{\xi(\sigma\eta-1) + \gamma\xi + \gamma(1-\xi)\sigma\zeta + \frac{\gamma}{1-\gamma}\sigma\eta\}\theta - \xi\sigma\psi\theta
\end{aligned}$$

where the variable θ is a constant function of the oil supply shock, which ensures balanced trade over the long-run. Under standard parameterizations,¹⁵ $\sigma\eta > 1$, and a decline in wealth lowers consumption and unambiguously raises GDP as in a standard real-business-cycle model. Thus, if a foreign, negative oil supply shock reduces domestic wealth, then we may observe a decline in consumption (if $c^{CM}(t) + \xi\theta < 0$), but the expansion of gross output (y) and GDP (l) in the complete markets model would be amplified, $y^{IM}(t) > y^{CM}(t)$ and $l^{IM}(t) > l^{CM}(t)$. Therefore, a negative oil supply shock is also expansionary at the zero lower bound in the standard new Keynesian model under incomplete markets.

¹⁵For example, Ferrero, Gertler, and Svensson (2008) set $\sigma = 1$ and $\eta = 2$, while Bodenstein, Erceg, and Guerrieri (2011) set $\sigma = 1$ and $\eta = 1.5$. Obstfeld and Rogoff (2005) argue that $\eta = 2$ is a reasonable calibration balancing micro and macro estimates. However, they also suggest that micro estimates, which imply larger values for η , are likely less biased. If we set $\eta \geq 2$, then any intertemporal elasticity of substitution $\sigma^{-1} \leq 2$ will satisfy this condition. Bayesian estimation of medium-scale macroeconomic models typically produce estimates in that range (e.g., Smets and Wouters, 2007).

G. Are oil supply shocks forecastable using oil futures?

I use the futures data available on the U.S. Energy Information Administration website,¹⁶ which provides 1, 2, 3, and 4-month crude oil future prices at daily frequency since 1983. The one-month contract expires on the third business day prior to the 25th calendar day of the month preceding the delivery month. If the 25th calendar day of the month is a non-business day, trading ceases on the third business day prior to the business day preceding the 25th calendar day. All subsequent contracts are for delivery on the months following the one-month contract. Thus, for each contract I use the price the day before trading ceases for the one-month contract.

I then construct changes in futures prices for the same delivery month. Let $p_{t,t+k}$ be the log price at time t for oil delivered at time $t+k$. The s -month change in the futures price for the same deliver month $t+k$ is given by,

$$\Delta^s p_{t,t+k} = p_{t,t+k} - p_{t-s,t+k}, \quad s = 1, \dots, 4-k, \quad k = 1, \dots, 3 \quad (13)$$

I then regress VAR-identified oil supply shocks on changes in oil price futures for that delivery month,

$$\text{oil}_t = \alpha + \beta \Delta^s p_{t-1,t} + \varepsilon_t, \quad s = 1, \dots, 2. \quad (14)$$

That is, I test whether past changes in oil futures can forecast today's oil supply shocks.

Table 1 reports the result for $s = 1, \dots, 3$. In all cases the coefficient β is small and insignificant. For example, in the first column a 1% increase in futures prices forecasts an (insignificant) -0.0035 standard deviation negative oil supply shock. Overall, very little of the variation in oil supply shocks appears to be forecastable using changes in oil price futures. This suggests that oil supply shocks I identify are unlikely to be confounded by anticipated demand shocks.

¹⁶http://www.eia.gov/dnav/pet/pet_pri_fut_s1_d.htm

Table 1 – Predictability of Oil Supply Shocks using Futures Prices

Dependent variable: Oil Supply Shock in the Following Month			
Futures Price Growth over past	1 month	2 months	3 months
	(1)	(2)	(3)
Growth of Futures Price	−0.0038 (0.0048)	−0.0040 (0.0027)	−0.00097 (0.0024)
R^2	0.002	0.005	0.001
Observations	366	386	364

Notes: The dependent variable is the oil supply shock in the following month. The independent variables is the change in the crude oil futures price over the past s months for crude oil delivery next month. Newey-West standard errors with 12-month bandwidth in parenthesis. + $p < 0.1$, * $p < 0.05$, ** $p < 0.01$.